Adaptive B-spline approach for inverse scattering problems

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1 Introduction

Various applications in field such as geophysical exploration, nondestructive evaluation and medical imaging are concerned with determining the location and the spatial variations of some physical properties of an object from measurements of the scattered response of a known electromagnetic or acoustic excitation. For example, in microwave imaging, the goal is to reconstruct the complex permittivity distribution of an object (the real part representing the permittivity and the imaginary part the conductivity). Several algorithms have been developed to solve this non linear and ill-posed problem. These algorithms must solve a large scale non linear optimization problem by generating values for all pixels even those which contain no useful information about the target. In this contribution, in order to reduce the problem complexity, an adaptive B-spline approach is proposed. B-splines have already show their interest for modelling curves and surfaces in computer aided geometric design (CAGD) due to theirs smoothness and localness properties [2, 3]. Moreover, they can be easily manipulated by dealing with knots. Our approach consists of approximating the object to be estimated using tensor product B-splines. The proposed inversion scheme is composed by sequences of non linear inversion, knot insertion and deletion process which allows us to simultaneously determine the knot distribution and to solve the inverse problem. Then, the inversion algorithm generate values only for interest area corresponding to the target and so the computational time is reduced. Moreover, the knot parameterization leads to reduce the freedom degree and to a natural shape-based representation and regularization. To fit with our problem, a specific knot insertion process, based on curvature information, and the deletion process, based on data fitting, are proposed. Moreover, to solve the non linear inverse problem step, within the B-spline approach, a conjugate gradient method [6] was used. Finally, to show the potentialities of this approach, some results dealing with simulated and experimental laboratory-controlled microwave data are presented.

2 Statement of the problem

A two-dimensional inhomogeneous obstacle embedded in a search domain Ω is irradiated successively by L transverse magnetic known incident fields e_l^{inc} $(l = 1, ..., L)$. The scattered field e_l^d is measured from M receivers distributed equidistantly around the object on the domain Γ. The object is characterized by its complex contrast function χ which is directly related to the dielectric permittivity and the conductivity of the object. By discretizing the search domain into a regular mesh $[N_x \times N_y]$ sized and using the method of moments [4], the two coupled integral equations of the forward problem can be written as two coupled matrix equations:

$$\mathbf{E}_l^{inc} = \mathbf{E}_l - \mathbf{G}_\Omega D(\chi)\mathbf{E}_l, \qquad \mathbf{E}_l^d = \mathbf{G}_\Gamma D(\mathbf{E}_l)\chi, \tag{1}$$

where $\mathbf{E}_l$ is the total field vector, $\mathbf{E}_l^{inc}$ is the incident field vector and χ is the contrast vector, all of dimension $N = N_x \times N_y$. $\mathbf{E}_l^d$ is the scattered field vector of dimension M and $\mathbf{G}_\Omega$ and $\mathbf{G}_\Gamma$ are the $[N \times N]$ and $[M \times N]$ matrix of the Green's functions, respectively. $D(\mathbf{t})$

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is a diagonal matrix whose entries are the elements of the vector $\mathbf{t}$.
These two coupled equations can be expressed as a non linear equation in χ:

$$\mathbf{E}_l^d = \mathbf{G}_\Gamma D \left[(\mathbf{I}_d - \mathbf{G}_\Omega D(\chi))^{-1} \mathbf{E}_l^{inc} \right] \chi, \tag{2}$$

where I_d denotes the identity matrix.

3 Adaptive B-spline inversion

3.1 Adaptive inversion process

The proposed approach consist to reduce the complexity of the inverse problem by approximating the object by tensor product B-spline basis (see [2] for an introduction to B-splines); then, the contrast is represented by using B-splines through knots instead of pixels. This leads to define the contrast as:

$$\chi = \mathbf{Ba} \tag{3}$$

where $\mathbf{B}$ is the matrix corresponding to the B-splines basis and $\mathbf{a}$ is a vector of expansion coefficients. Thus, the problem can be reformulated as:

$$\mathbf{E}_l^d = \mathbf{G}_\Gamma D \left[(\mathbf{I}_d - \mathbf{G}_\Omega D(\mathbf{Ba}))^{-1} \mathbf{E}_l^{inc} \right] \mathbf{Ba}. \tag{4}$$

The inversion problem of reconstructing the permittivity contrast χ, of dimension N, is then translated as the reconstruction of an auxiliary vector $\mathbf{a}$ of dimension depending on the number of knots but smaller than N. The corresponding non linear inversion is then solved by minimizing, using a conjugate gradient method approach [6], the following cost functional:

$$J = \sum_{l=1}^{L} \left\| \mathbf{E}_l^d - \mathbf{G}_\Gamma D \left[(\mathbf{I}_d - \mathbf{G}_\Omega D(\mathbf{Ba}))^{-1} \mathbf{E}_l^{inc} \right] \mathbf{Ba} \right\|^2. \tag{5}$$

The problem is now to determine the better knot distribution which describe with the few knots the object. In order to solve the inversion problem and to find the best knot distribution, an adaptive scheme is proposed. The goal being to refine the knot distribution while doing the reconstruction. The steps of this procedure can be summarized as follows:

- Starting from a distribution of few knots, which do not take into account *a priori* on the target position, a first non linear inversion is done.
- From this first estimate, the following process is incremented:
  - insert new knots at “interest” area
  - delete the redundant or ineffective knots
  - solve the corresponding non linear problem

As the first distribution of knots can be seen as a coarse representation scale, few iterations in the inversion process are necessary. In this work the number of maximum iterations for each non linear inversion is fixed as a parameter at the begin of the algorithm to nb_iter. The process is stopped when our insertion process can not insert new knots, when the criterion is smaller than a determined value or when the maximum number of iterations (fixed at the beginning of the algorithm) is reached.

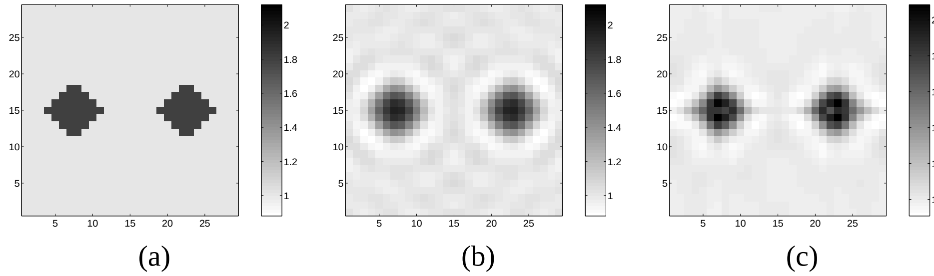

Figure 1: (a) simulated object, (b) reconstruction obtained using the conjugate gradient algorithm, (c) reconstruction obtained using the adaptive approach.

3.2 Knot insertion

Given a current estimate of the object, the insertion step is designed to allow for the refinement of the representation in what are hypothesized to be “interesting” areas. Drawing on related efforts in the CAGD community,“interesting” in this case can be defined as regions of high mean curvature (see [3] for computational details). From the current estimated object the corresponding spline representation is determined and a normalized curvature map is constructed. Then, a threshold Th, based on a maximum value percentage, is introduced in order to determine the ”interesting” area and so the position of the new knots.
One can note that if the value threshold is chosen too small, this leads to introduce redundant or ineffective knot and so to increase the deletion computational time step.

3.3 Knot deletion

After each insertion process, a deletion step is done in order to delete the redundant or inefficient knots. The goal is to remove knots in a way that does the least damage to the accuracy of the reconstruction as measured by the cost function. Here we do in fact refer back to the data. In this way, the computation of a weight for each knot is done through a ”linearized” criterion value by considering the total field constant during the process. This is done in order to speed up the process. The deletion process can be summarized as follows: compute the weight (associated to the criterion value) of each knots by removing the considered knot from the distribution, then removed the knots with few impact. As the weight of the neighbor knots of a suppressed one change, the process is done until no knot can be deleted.

4 Numerical results

The considered configuration consists of two distinct square dielectric homogeneous cylinders [5] of diameter $4\lambda/5$, separated by a distance of approximatively $\lambda/2$. The contrast of these cylinders is $\chi = 1.8$. The search domain, of dimension $d = 3\lambda$, is discretized into 29×29. Moreover, $L = 29$ electromagnetic excitations and $M = 29$ receivers have been considered. In order to initialize the algorithm, the following knot distribution has been taken into account: k_x=[1, 7, 14, 21, 29] and k_y=[1, 7, 14, 21, 29], the threshold was fixed to $Th = 85\%$ and a maximal number of iterations for the non linear inversion was fixed to 5 for the first one and to 10 for the others.
Figure 1 presents the reconstructions obtained after 45 iterations using the conjugate gradient algorithm and the adaptive B-spline approach (4 sequences of non linear inversion and 3 insertion-deletion processes). These results show an enhancement of the final solution, particularly for the shape, using the adaptive approach. The final knot distribution is composed

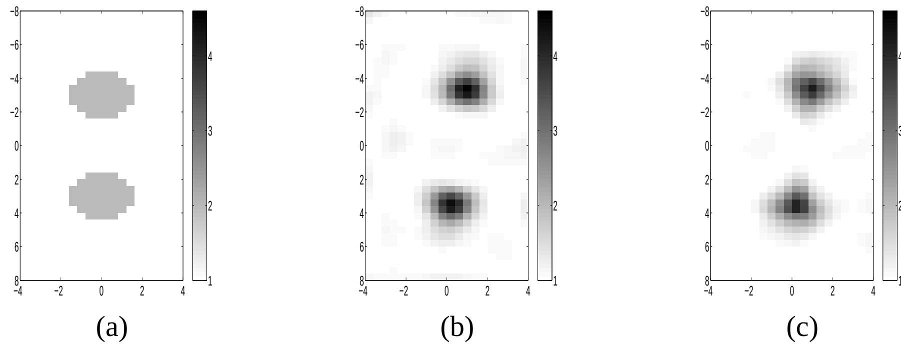

Figure 2: (a) grey map level of the simulated experimental object. Reconstruction at 4 GHz obtained using (b) a conjugate gradient algorithm and (c) using the adaptive approach after 55 iterations.

by: k_x=[1, 5, 5.59, 6.55, 7.77, 9.33, 14, 18.66, 20.22, 21, 21.88, 23.66, 29] and k_y=[1, 7, 11.66, 12.70, 13.22, 14, 14.51, 15.29, 16.33, 21, 29]. Then, the final estimated object is represented by 143 control points through 24 knots in the B-spline approach instead of 841 pixels.

5 Real data

The considered experimental setup, from Institut Fresnel (Marseille - France), is detailed in [1]. In this contribution, the reconstructions obtained for a dielectric target made of two identical cylinders with circular cross-section of radius 1.5 cm are presented. The relative permittivity of the cylinders has been estimated to $\varepsilon_r = 3 \pm 0.3$. Here, the data associated to the so called 'twodielTM_8f.exp' file for the 4 GHz frequency are used.
Figure 2.a shows the ideal target in a rectangular centered search domain of 8 cm (along the x-axis) $\times 16$ cm (along the y-axis), discretized into 20×40 cells. Figure 2.b and 2.c present the reconstructions obtained after 55 iterations using a conjugate gradient method and the adaptive B-spline approach, respectively. The final estimated object is represented by 117 elements in the B-spline approach instead of 800 elements in the pixelled approach. The shift of the cylinder centers is within the experimental margin.

6 Conclusion

In this contribution, to solve a two-dimensional inverse scattering problem, an adaptive B-spline approach was proposed in order to reduce the problem complexity. This algorithm permits to simultaneously determine a knot distribution, using a specific knot insertion-deletion process, and to solve the inverse problem. The results, obtained using simulated and real data, show the potential and the interest of the proposed approach.

References

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