Precorrected-FFT Solution of the Volume Integral Equations for Inhomogeneous Dielectric Bodies

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Abstract: The precorrected-FFT method is applied to the fast solution of the volume integral equation for lossy, inhomogeneous dielectric bodies. The volume of the dielectric body is discretized into tetrahedron elements and the SWG basis functions are employed to expand the unknown electric flux density. The basis functions are then projected onto a uniform grid surrounding the nouniform mesh, enabling the FFTs to be used to speed up the matrix-vector multiplies in the iterative solution of the matrix equation. The resultant method has a computational complexity and memory requirement of $O(N \log N)$ and $O(N)$ respectively.

1. INTRODUCTION

The volume integral equation (VIE) in conjunction with the method of moments (MoM) is a powerful tool for the analysis of electromagnetic scattering from dielectric bodies of arbitrary shape and inhomogeneity. However, the traditional MoM suffers from tremendously high computational cost and memory requirement as the electrical size of the scatterers increases. Recent development in fast algorithms has alleviated this problem to some extent. The most widely used approach to solve the VIE is the conjugate gradient fast Fourier transform (CGFFT) [1]. This method requires the volume of the object to be discretized into uniform hexahedral cells in order to use the Toeplitz property of the coefficient matrix. Thus when modeling an arbitrary geometry, very dense cells are required which will result in a large number of unknowns and the unavoidable staircase geometry error will degrade the accuracy of the final solution. To overcome this drawback, the multilevel fast multipole algorithm (MLFMA) has been applied to solve the hybrid volume-surface integral equation (VSIE) for composite conducting and dielectric objects [2]. In this paper, we apply a precorrected-FFT based algorithm to efficiently solve the volume integral equation. Unlike the CGFFT, the present approach uses the more flexible tetrahedral mesh to model the object. Then, the entire object is enclosed in a uniform rectangular grid and all the basis functions are projected onto the surrounding grid points. By such a procedure, the interactions between the point sources on the grid points are described by three-dimensional convolutions which can be computed rapidly by FFTs. In this technique, the density of the uniform grid depends on the requirement of the solution accuracy and can retain a desired coarse level even if a complex structure is analyzed. This technique offers good flexibility to model arbitrarily shaped structures while keeping the efficiency of the FFTs.

2. FORMULATION

2.1 The Formulation and Discretization of the Volume Integral Equation

Consider a lossy, inhomogeneous dielectric body V illuminated by an incident field $\boldsymbol{E}^i$. Assume that the material is dielectric ($\mu = \mu_0$) and has complex dielectric constant of $\varepsilon(\boldsymbol{r}) = \varepsilon_r(\boldsymbol{r})\varepsilon_0 - j\sigma(\boldsymbol{r})/\omega$, where ε_r and σ are the relative permittivity and conductivity at position $\boldsymbol{r}$. By invoking the equivalence principle, the dielectric body is removed and replaced by a volume polarization current $\boldsymbol{J}$. According to the fact that the total electric field

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is the sum of the incident field and the scattered field due to $\boldsymbol{J}$, we can obtain the following volume integral equation,

$$\boldsymbol{D}(\boldsymbol{r})/\varepsilon(\boldsymbol{r}) = \boldsymbol{E}^{i}(\boldsymbol{r}) - j\omega\boldsymbol{A}(\boldsymbol{r}) - \nabla\Phi(\boldsymbol{r}) \tag{1}$$

where $\boldsymbol{A}(\boldsymbol{r})$ and $\Phi(\boldsymbol{r})$ are the vector and scalar potentials produced by the volume current $\boldsymbol{J}$, and $\boldsymbol{J}$ is related to the total electric flux density by $\boldsymbol{J}(\boldsymbol{r}) = j\omega(\varepsilon(\boldsymbol{r}) - \varepsilon_0)\boldsymbol{D}(\boldsymbol{r})/\varepsilon(\boldsymbol{r})$.

To solve Eqn. (1), the volume V is discretized into a number of tetrahedral elements, in each of which the dielectric properties are approximated as constant. The unknown electric flux density can be represented by the volumetric SWG basis functions [3]:

$$\boldsymbol{D}(\boldsymbol{r}) = \sum_{n=1}^{N} D_n \boldsymbol{f}_n(\boldsymbol{r}) \tag{2}$$

where D_n represent the unknown expansion coefficients, N denotes the number of faces that make up the tetrahedral model of V. Substituting (2) into (1) and applying Galerkin's testing procedure yield a $N \times N$ matrix equation of the form

$$\boldsymbol{S}\boldsymbol{D} = \boldsymbol{E} \tag{3}$$

The elements of the coefficient matrix $\boldsymbol{S}$ and the excitation vector $\boldsymbol{E}$ can be derived from (1) and the expressions are omitted here. But for easier description of the following P-FFT approach, we give the expressions of the contributions to $\boldsymbol{A}$ and Φ from a single basis function, which are needed in the computation of the elements of $\boldsymbol{S}$,

$$\boldsymbol{A}_n(\boldsymbol{r}) = \frac{\mu_0 a_n}{12\pi}\left[\frac{\kappa_n^+}{V_n^+}\int_{T_n^+}\boldsymbol{\rho}_n^+(\boldsymbol{r}')\frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}dv' + \frac{\kappa_n^-}{V_n^-}\int_{T_n^-}\boldsymbol{\rho}_n^-(\boldsymbol{r}')\frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}dv'\right] \tag{4}$$

$$\Phi_n(\boldsymbol{r}) = \frac{-a_n}{4\pi j\omega\varepsilon_0}\left[\frac{\kappa_n^+}{V_n^+}\int_{T_n^+}\frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}dv' - \frac{\kappa_n^-}{V_n^-}\int_{T_n^-}\frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}dv' - \frac{(\kappa_n^+ - \kappa_n^-)}{a_n}\int_{a_n}\frac{e^{-jk_0|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}ds'\right] \tag{5}$$

The definitions of a_n, $\boldsymbol{\rho}_n^{\pm}$, $V_n^{\pm}$ and $\kappa_n^{\pm}$ can be found in [3]. Note that the third term in Eqn. (5) corresponds to surface charges which only exist on the interfaces of different media.

2.2 The Precorrected-FFT Solution of the VIE

The precorrected-FFT method has been applied to solve the surface integral equations for electromagnetic scattering problems [4]. It can also be applied to the solution of the volume integral equations with some modifications. The method separately considers near- and far-field interactions when evaluating a matrix-vector multiplication. To compute far-field interactions, sources supported by the scatterer are projected onto a regular grid by matching their vector and scalar potentials at some given test points to guarantee the approximate equality of their far fields. Next, the potentials (fields) at other grid locations produced by these grid-projected sources are evaluated by a 3-D convolution. Knowledge of these fields permits the computation of fields on the scatterer through interpolation. The projection and interpolation operators are represented by sparse matrices, while the convolution can be effected using an FFT. Unfortunately, the near fields radiated by these grid currents do not match those radiated by the original sources. Therefore, near-field interactions are evaluated directly, and corrected for errors introduced by the far-field operator. The implementation of the projection step for the VIE will be described in the following paragraph and the convolution, interpolation, and precorrection steps are omitted due to space limitation since they are similar to those for the SIE, although more complicated.

To implement the P-FFT method, the entire object is enclosed in a uniform rectangular grid which is further subdivided into small cells with each cell consisting of p^3 grid points and containing only a few tetrahedral elements. Assume the nth volumetric SWG basis function $\boldsymbol{f}_n$ is contained in a given cell k. For the projection of the electric charges (corresponding to

$\nabla \cdot f_n$), enforcing the scalar potential produced by the electric charges at the p^3 grid points to match that produced by the original electric charge distributions on the two tetrahedral elements and the common triangular patches (if applicable) at N_c test points, we can obtain the projection operator for the divergence operator of the *n*th basis function

$$W(k,n) = \left[\boldsymbol{P}^{gt}\right]^{+} \boldsymbol{P}^{pt,n} \tag{6}$$

where $\boldsymbol{P}^{pt,n}$ denotes the *n*th column of $\boldsymbol{P}^{pt}$ and $\left[\boldsymbol{P}^{gt}\right]^{+}$ indicates the generalized inverse of $\boldsymbol{P}^{gt}$. $\boldsymbol{P}^{gt}$ represent the mappings between the grid charges and the test-point potentials and $\boldsymbol{P}^{pt}$ represent the mappings between the actual charge distributions and the test-point potentials, respectively, given by

$$\boldsymbol{P}^{gt}(q,l) = \frac{1}{4\pi\varepsilon_0} \frac{e^{-jk_0\left|\boldsymbol{r}_q^t - \boldsymbol{r}_l\right|}}{\left|\boldsymbol{r}_q^t - \boldsymbol{r}_l\right|} \tag{7}$$

$$\boldsymbol{P}^{pt}(q,n) = \frac{-a_n}{4\pi j\omega\varepsilon_0}\left[\frac{\kappa_n^+}{V_n^+}\int_{T_n^+}\frac{e^{-jk_0\left|\boldsymbol{r}_q^t - \boldsymbol{r}'\right|}}{\left|\boldsymbol{r}_q^t - \boldsymbol{r}'\right|}dv' - \frac{\kappa_n^-}{V_n^-}\int_{T_n^-}\frac{e^{-jk_0\left|\boldsymbol{r}_q^t - \boldsymbol{r}'\right|}}{\left|\boldsymbol{r}_q^t - \boldsymbol{r}'\right|}dv' - \frac{\left(\kappa_n^+ - \kappa_n^-\right)}{a_n}\int_{a_n}\frac{e^{-jk_0\left|\boldsymbol{r}_q^t - \boldsymbol{r}'\right|}}{\left|\boldsymbol{r}_q^t - \boldsymbol{r}'\right|}ds'\right] \tag{8}$$

where $\boldsymbol{r}_q^t$ and $\boldsymbol{r}_l$ are the position vectors at the *q*th test point and the *l*th grid point, respectively, and $\hat{q}_l$ is the charge at the *l*th grid point. For any basis function n in cell k, this projection operator generates a subset of the grid currents $\bar{q}$. The contribution to $\hat{q}$ from the charges in cell k can be computed by summing over all the actual charges in this cell, *i.e.*

$$\bar{q} = \sum_n \boldsymbol{W}(k,n) D_n \tag{9}$$

Following the above procedure, we can project the charges $D_n \nabla \cdot \boldsymbol{f}_n$ onto the p^3 grid points surrounding cell k. It should be noted that the projection of the volume and surface charges are performed simultaneously in one step, which is a convenient and efficient scheme developed for the volume integral equation. Similarly, by matching the vector potential due to the p^3 grid currents and that due to the actual volume current distributions at the test points, we can obtain the projection operators for the electric currents.

3. NUMERICAL RESULTS

Consider a $5\lambda_0 \times 1\lambda_0 \times 0.6\lambda_0$ rectangular dielectric box with a relative dielectric constant of $\varepsilon_r = 1.75 - j0.3$. The volume is discretized into 18,000 tetrahedrons, leading to 37,720 unknowns. The bistatic RCS obtained by the P-FFT method in conjunction with the VIE and SIE respectively are shown in Fig.1 and are also compared with the AIM solution of the SIE [5]. Good agreements are observed for both polarizations, validating the present method.

The second example is another rectangular dielectric box with the size of $3.5\lambda_0 \times 2.0\lambda_0 \times 0.25\lambda_0$ and the dielectric constant of $\varepsilon_r = 3. - j0.09$. The volume is divided into 18,000 tetrahedrons, yielding 37,720 unknowns. The monostatic RCS $\sigma_{\theta\theta}$ calculated by the P-FFT method based on the VIE and SIE are compared in Fig.2 and the iteration numbers versus the incident angles are also shown in Fig.3. It is observed that number of iterations required to achieve a 10^{-3} residual for the VIE are only a quarter of that for the SIE.

4. CONCLUSIONS

The precorrected-FFT method has been applied to solve the volume integral equation for the scattering from dielectric objects with arbitrary shape and inhomogeneity. The application

of the P-FFT significantly reduces the memory requirement and computational complexity of the MoM solution of the VIE. Although tetrahedral elements are used to discretize the dielectric volume in this paper, other elements such as the curvilinear tetrahedron can also be used without complex modifications, which makes the present method a powerful tool for the solution of the VIE.

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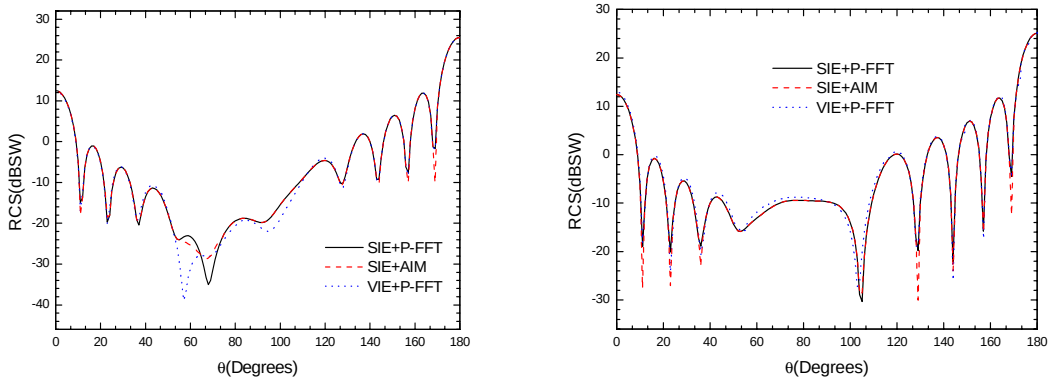

(a) $\theta\theta$ polarization (b) $\phi\phi$ polarization

Fig.1. Bistatic RCS of a $5\lambda_0 \times 1\lambda_0 \times 0.6\lambda_0$ dielectric rectangular box

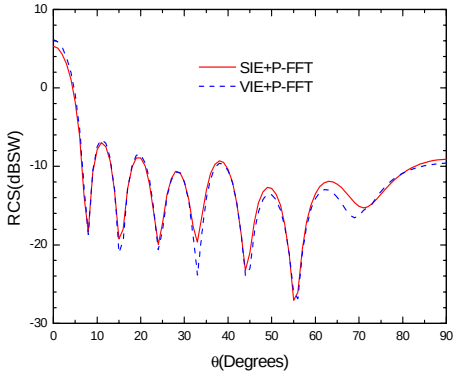

Fig.2 Monostatic RCS $\sigma_{\theta\theta}$ of a $3.5\lambda_0 \times 2.0\lambda_0 \times 0.25\lambda_0$ dielectric rectangular box

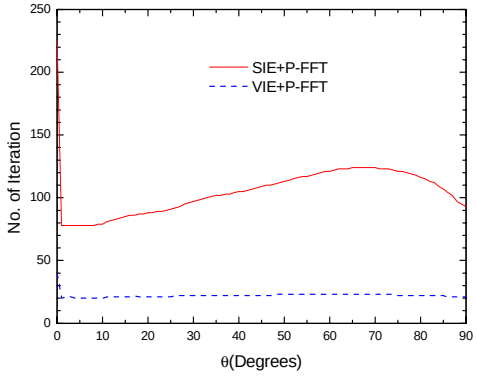

Fig.3 Iteration numbers needed by the GMRES solution of the VIE and SIE