ON THE CONDITION NUMBER OF VARIOUS FINITE ELEMENT MATRICES INVOLVED IN THE NUMERICAL SOLUTION OF ELECTROMAGNETIC RADIATION OR SCATTERING PROBLEMS

Bruno Stupfel
CEA/CESTA, Commissariat à l'Énergie Atomique, B.P. 2, 33114 Le Barp, France

The finite element method (FEM) is a powerful numerical technique for solving scattering problems involving inhomogeneous penetrable 3D objects. For open region problems, the radiation condition may be rigorously taken into account by prescribing either an exact radiation condition (e.g., an integral equation) on the outer surface of the object, or approximately by implementing an absorbing boundary condition (ABC) or a perfectly matched layer (PML) on the outer boundary terminating the FE mesh. The whole set of equations may be solved either directly, or iteratively by using, e.g., a domain decomposition method that reduces the memory storage requirements.

All of these techniques imply the solution of one or several linear systems that result from the FE discretization of Maxwell's equations inside a given (here simply connex) computational domain, on the boundary of which various boundary conditions (BC's) are prescribed. If the electrical size of this domain is large, then the number of unknowns may be such that iterative solvers are needed to reduce the computing time and memory storage. In this case, the number of iterations and, hence, the computational time required to achieve a given numerical accuracy are known to increase with the condition number κ of the FE matrix.

In this paper, we attempt to draw the rules that govern the behaviour of κ. To this effect, an eigenmode technique is proposed that allows us to dissociate the influence of the FE mesh and FE bases fuctions from the one of the actual physical cavity, i.e., the computational domain with its constitutive materials and BC's. We introduce the eigenmodes associated with this cavity, and establish the relationship between the eigenvalues of the FE matrix and those of the matrix of the same FE variational formulation written in the basis of these eigenmodes. Numerical examples are provided for 1D and 3D problems that illustrate the results so obtained. For example, κ increases with the number of unknowns N; for fixed N, it increases when the order of the FE bases functions increases, or when the frequency decreases. Also, regarding the PML, we have come to the conclusion that the ill-conditioning of the FE matrices reported in the literature does not stem from the characteristics of the physical cavity, but from the FE mesh indeed.