Solution of Electromagnetic Boundary Value Problems by the Plane Wave Enriched Finite Element Approach

Mustafa Kuzuoglu (*)
Department of Electrical Engineering
Middle East Technical University
06531, Ankara TURKEY

Raj Mittra
EM Communication Laboratory
319 EE East, Pennsylvania State University
University Park, PA 16802 USA

The numerical solution of electromagnetic boundary value problems by the Finite Element Method (FEM) yields reliable results with a moderate number of degrees of freedom for low and medium frequencies when the well-known 'resolution rule' is satisfied. According to this rule, for an accuracy of 1 %, around 10 nodes per wavelength are required. Therefore, if the wavelength is small with respect to the physical dimensions of the computational domain, the matrix resulting from the FEM discretization becomes huge especially for 3D problems. In this case, the problem becomes unmanageable even with very powerful computers.

In this paper, we present a new approach for the FEM solution of high frequency problems by defining new basis functions for the representation of field quantities. The basic idea is to incorporate the known behavior of the solution in a rational way to improve the performance of standard FEM models. To this end, the finite element space is constructed by multiplying the standard shape functions with a space of functions having good local approximation property. This seems to be a natural way to include 'a priori' information about the local behavior of the solution.

In 2D applications, the finite element basis functions are obtained by multiplying the standard finite element shape functions N_i by a set of functions $p_m(\vec{r}) = \exp(-jk\hat{u}_m.\vec{r})$, which represent plane waves propagating in the direction of the unit vector $\hat{u}_m$. The standard basis functions are 'enriched' by the plane wave solutions. It should also be mentioned that each N_i, having a compact support, acts as the 'windowing function' of the plane wave solutions. In this way the conformity of the FEM formulation is preserved. In 2D, we seek a solution to the Helmholtz equation of the form

$$\varphi(\vec{r}) = \sum_{i=1}^{N}\sum_{m=1}^{M} \varphi_{im} N_i(\vec{r}) \exp(-jk\hat{u}_m.\vec{r})$$

This expression is substituted in the weak variational formulation to yield the FEM matrix equation. The elements are no longer subject to the 'resolution rule', and it is possible to handle high frequency problems by a relatively reduced number of unknowns. A higher order Gaussian quadrature is used to evaluate the oscillatory integrals appearing in the calculation of the FEM matrix entries. In addition, owing to the special form of the FEM representation, PML mesh truncation is successfully applied in the plane wave enriched FEM formulation for the truncation of the unbounded computational domain.

In spite of the increased number of degrees of freedom per node, this approach results in a considerable decrease in the computational burden, realized by representing the fields in the domain by a reduced number of 'electrically large' elements.