On the Application of Wavelet-Like Basis Functions in Finite Element Algorithms

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Wavelets and wavelet techniques have become important topics in the computational sciences in recent years. Their application in the area of research in computational electromagnetics has only occurred rather recently; however, wavelets have received abundant attention in the scientific literature. Wavelets have been used in many different algorithms. For example, wavelet methods for differential equations have been developed by Jaffard and Laurencot (S. Jaffard and Ph. Laurencot, Wavelets: A Tutorial in Theory and Applications, pp. 543-601, 1992). The use of wavelet-like basis functions in the finite element solution of one-dimensional electrostatics problems in which either Dirichlet or Neumann boundary conditions are enforced at each endpoint of the interval has been discussed by Gordon (R. Gordon, *Proc. of the 11th Annual Review of Progress in Applied Computational Electromagnetics*, pp.559-567, 1995). Although the methods discussed above were using wavelets in frequency domain approaches, they have also been used in time domain techniques. Krumpholz and Katehi have used wavelet expansions in the multiresolution time domain (MRTD) method (M. Krumpholdz and L. P. B. Katehi, *IEEE Trans. on Microwave Theory and Techniques*, vol. 44, no. 4, pp. 555-571, April 1996).

In this presentation, wavelet-like basis functions will be incorporated into several different differential equation based algorithms. A two-dimensional time domain algorithm employing wavelet-like basis functions will be used to solve electromagnetics problems. One difficulty with the traditional FETD method is that it can require the solution of a large matrix equation for each time step; however, in the time domain approach presented here, the solution of a matrix equation for each time step is not required. In this sense, it is more like the traditional FDTD technique originally developed by Yee (Yee, *IEEE Trans. Antennas Prop.*, vol. AP-14, pp. 302-207, 1966). To generate the wavelet-like functions, a variation of the technique presented by Jaffard will be used. Following Jaffard, Hutchcraft, Harrison, and Gordon have previously generated two-dimensional wavelet-like basis functions by beginning with the traditional tetrahedral basis functions. But in this presentation, instead of pursuing this more time consuming approach, a combination of one-dimensional wavelet-like basis functions will be used to generate the two-dimensional basis. First, Neumann and Dirichlet basis functions will be developed for both the x and y directions. For each field component, the two-dimensional basis functions will be derived from combinations of two sets of the one-dimensional basis functions. Finally, these new basis functions are employed in an FETD algorithm. This method will be used for the solution of several problems; comparisons with analytical and FDTD results will be presented. Also, two-dimensional electrostatics problems will be solved using the same technique described above to derive the two-dimensional basis set. Comparisons of the number of steps required for convergence will be made between the traditional finite element algorithm, the previous method for developing wavelet-like basis functions, and the new method for generating two-dimensional wavelet-like basis functions. Finally, higher order wavelet-like basis functions will be discussed. These basis functions are generated from the higher order finite element basis functions. Comparisons between first order and higher order traditional and wavelet-like basis functions will be presented.