An $O(N^{3/2})$ MoM Computation of Electromagnetic Radiation and Scattering Problems Using A Novel Single Level IE-SVD Algorithm

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Solving large-scale electromagnetic problems by integral equation methods suffers from $O(N^2)$ numerical complexity of memory requirement and CPU time, where N is the number of basis functions. A novel single level dual rank IE-SVD algorithm efficiently compresses the system matrix to reduce the memory requirement and CPU time for both matrix assembly and matrix-vector multiplication to $O(N^{3/2})$. The algorithm forms the $Q-R$ factorization of rank deficient local matrices due to non-self group interactions by ranking the most linearly independent basis functions in the transmitting group and the most significant ones in the receiving group. There have been a number of successful algorithms that reduce the numerical complexity of IE methods such as the fast multipole method (FMM) [1] (R. Coifman, V. Rokhlin, and S. Wandzura, *IEEE AP. Mag*, 7-12, 1993) and the SVD based algorithms presented in [2] (S. Kapur and J. Zhao, *DAC*, 141-146, 1997) and [3] (S. Kapur and D. E. Long, *ICCAD*, 448-455, 1997). The 2-level FMM combined with the iterative techniques has reduced the numerical complexity to $O(N^{3/2})$ to solve dense integral equation matrices that arise from the Helmholtz equation. One major drawback of this approach is its dependence on the integral equation kernel. For complex Green's functions, such as the layered medium Green's function, the application of FMM is much more involved than the surface scattering problems where the free space Green's function can be employed. The SVD algorithm presented in [2] reduces the numerical complexity of the matrix-vector multiplication to $O(N \log N)$, but its matrix assembly stage, which requires the construction of the impedance matrix prior to its factorization, has $O(N^2)$ computational cost. The integral equation solver IES^3 given in [3] reduces the numerical complexity of both the matrix assembly and solution to $O(N \log N)$ by using an adaptive grouping of basis functions and an interpolation-based construction of the $Q-R$ matrices in SVD, which assumes that the integral equation kernel is smooth over local regions of space. The key step in [3], the matrix assembly process without the knowledge of the full MoM matrix, is very vague. The method presented in this paper tends to overcome the above drawbacks of the previous algorithms such that: It is independent of the integral equation kernel, or it does not depend on the exact knowledge of the Green's function; It assembles the local Q and R matrices without a priori knowledge of the local matrix itself; It is free of the previous assumptions about the kernel of the integral equation, namely, the smoothness of the kernel to implement reduced sampling procedure in [3].