Multilevel Non-uniform Grid Algorithm for Fast Iterative Solution of Scattering Problems

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Rigorous analysis of scattering by arbitrary shaped bodies is often effected via numerical solution of pertinent integral equations. For simplicity, we consider a two-dimensional scattering by an open perfectly conducting surface analyzed via the Electric Field Integral Equation (EFIE). The number of field and current sampling points on the surface is proportional to its electrical dimensions, i.e., of $O(N)$, where $N = kR$, R being the radius of the smallest circle circumscribing the scatterer and k - the wavenumber. Due to $O(N^3)$ computational complexity of conventional direct solvers, iterative approach becomes a necessity for analysis of electrically large problems. Solving EFIE, each iteration requires evaluation of the electric field due to a given surface current. Straightforward evaluation of the field at $O(N)$ points by surface integration involving summation of $O(N)$ terms amounts to $O(N^2)$ operations. This high computational burden underlines the need for using fast field evaluation techniques.

In this paper, we develop a novel non-uniform grid (NG) technique that facilitates the numerically efficient evaluation of the field produced by a given current distribution (A. Boag, et al., *IEEE Antennas Wireless Propagat. Lett.*, vol. 1, no. 7, pp. 142-145, 2002). The algorithm is based on the observation that, locally, the phase compensated field radiated by a finite size source is an essentially bandlimited function of the angular and radial coordinates of the source centered spherical coordinate system. The angular bandwidth is proportional to the linear dimensions of the source, while the local bandwidth with respect to the radial distance decreases rapidly with the distance. Therefore, the radiated field can be sampled on a non-uniform spherical grid with radial density decreasing with the distance from the source. The total number of grid points is proportional to the source region dimensions.

With this in mind, we introduce a multilevel algorithm based upon a hierarchical decomposition of the scatterer surface into subdomains. At each level, the domain decomposition proceeds by subdividing each parent domain into a number of subdomains. At the finest level, for each small subdomain, radiated fields are computed directly at a small number of points of a very coarse non-uniform spherical grid. Next, the fields of each group of adjacent subdomains are aggregated into those of the parent subdomain and defined over a finer non-uniform grid. Transition from coarse to fine grids is effected by interpolation. The phase common to all source points in a given subdomain is removed from the field prior to interpolation, thus rendering the field a slowly varying function of the spatial variables. Following the interpolation step, the phases are restored and partial fields aggregated. At the highest level, the fields are interpolated to the scatterer surface. The proposed multilevel algorithm attains an asymptotic complexity of $O(N \log N)$. A multilevel implementation of the NGTD scheme will be presented along with numerical examples demonstrating its efficacy.