Integral equation methods for solving problems of scattering on an unclosed cone structure

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The method based on using the integral Kontorovich-Lebedev transforms and the semi-inversion method is a successful one for studying boundary electrodynamics problems with single periodical slotted cone geometry(there is one cone strip in the period). Taking into consideration a multi-element cone structure(there are several strips in the period) makes applying the above mentioned method extremely tedious. The task of this work is to find a method that uses singular integral equation to solve an excitation problem for a periodic cone structure with any number of strips(S) in the period.

Let a semi-infinite perfectly conducting infinitely thin circular cone Σ with N equally spaced slots cut along the generatricies be excited by a radial harmonic electric or magnetic dipole. The angular width of the slots and the period of the cone are d and $l = 2p/N$ respectively. By virtue of introducing Debye potentials and the integral Kontorovich-Lebedev transforms the electrodynamics boundary problem is reduced to solving dual series equations for unknown Fourier coefficients $\hat{y}^{(j)}_{m,n}$ of electromagnetic field components. The dual series equations are equivalent to singular integral equations like these:

1) for electrical dipole excitation, $j=1$,

$$\frac{1}{2p}\int_S \hat{F}(y-a)\Phi_1(a)\,da + \frac{1}{2p}\int_S \left[\hat{A}^{(1)}_{mt} - \hat{K}_1(y-a)\right]\Phi_1(a)\,da = e^{im_0 y},$$

$$y \in S : \frac{pd}{l} < |y| \le p, \quad \hat{F}(y-a) = \sum_{n\neq 0} \frac{1}{N(n+n)}\frac{|n|}{n} e^{in(y-a)},$$

$$\hat{K}_1(y-a) = \sum_{n\neq 0} \frac{1}{N(n+n)}\frac{|n|}{n}\hat{e}^{(1)}_{m,n} e^{in(y-a)};$$

2) for magnetic dipole excitation, $j=2$,

$$\frac{1}{p}\int_S \frac{\Phi_2(x)}{x-y_1}\,dx + \frac{1}{p}\int_S K(x-y_1)\Phi_2(x)\,dx = ie^{im_0 y_1}, \; y_1 \in S,$$

$$K(x-y_1) = \frac{1}{2}ctg\frac{x-y_1}{2} - \frac{1}{x-y_1} - \frac{i}{2N}\left(\frac{pe^{inx}}{\sin pn} - \frac{1}{n}\right)\frac{1}{A^{(2)}_{mt}} - \frac{i}{2}\sum_{n\neq 0}\frac{|n|}{n}\hat{e}^{(2)}_{m,n} e^{in(y_1-x)};$$

$$\Phi_j(y) = \sum_{n=-\infty}^{+\infty} \hat{y}^{(j)}_{m,n} e^{iny}, \quad y \in [-p, p], \quad \hat{e}^{(j)}_{m,n} = O\left(\frac{\sin^2 g}{N^2 (n+n)^2}\right), \; -1/2 \le n < 1/2,$$

$m, m_0 \in Z$, $\hat{A}^{(1)}_{mt}$, $A^{(2)}_{mt}$ are known.

These integral equations can be solved numerically by the descrete singularities method.