FAST COMPUTATION AND EXTRAPOLATION OF THE EFFECTS OF ARRAY TRUNCATION IN BROADBAND ANTENNA ARRAYS

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Abstract: In broadband phased arrays, the elements near the edges have a widely deviating behavior. This is analyzed here with the help of a finite-by-infinite array approach. The solution of the equations is dramatically accelerated by decomposing the array into two approximate semi-infinite arrays (APSIA's), the solution of which is obtained recursively for arrays of increasing sizes. For very large arrays, the solution is extrapolated further inside the array by using a model for the waves scattered by the edges of the array. Simulations results are shown for arrays of tapered-slot antennas in receiving conditions.

1 Introduction

Given the very strong couplings between the elements of broadband phased arrays, elements near the edges present a widely deviating behavior with respect to the infinite-array solution. If the array to be designed is rectangular, a good way of obtaining an estimate of the effects of array truncation consists of simulating finite-by-infinite arrays. For large arrays made of element having complex shapes, the solution of the finite-by-infinite array problem may still require a prohibitive computation time.

This paper is concerned with a fast technique to solve the finite-by-infinite array equation system. This is done by decomposing the array into two approximate semi-infinite arrays (recalled in Section 2). The major advantage of this representation is that the difference w.r.t. the infinite-array solution may be regarded as a single wave scattered from the unique edge of the array. The technique presented here consists of following this wave in a recursive way to solve problems involving an increasing number of antennas in the finite-array direction (Section 3). When this number becomes very large, the edge-scattered waves can also be efficiently extrapolated (Section 3). Simulation results are shown in Section 4, and conclusions are drawn in Section 5.

2 Decomposition into approximate semi-infinite arrays

The array structure to be simulated is depicted in Fig. 1. The example shown here consists of an array of tapered-slot antennas which can be electrically connected to each other. The array is finite in the X direction, and infinite in the Y direction. As shown in Fig. 2, such an array can be decomposed into two approximate semi-infinite arrays (APSIA's). The connection between this decomposition and the fringe integral equation approach [2] is described in [1]. The term "approximate" refers to the fact that the currents induced on antennas $N+1$ to ∞ are assumed to correspond to the infinite-array solution. Despite this approximation, the equation depicted in Fig. 2 is exact. Hence, if a very fast solution can be found for the left (A) and right (B) APSIA's, the finite-by-infinite array solution can be solved exactly. The technique

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presented below for the solution of the APSIA problem is based on the properties of the difference with respect to the infinite-array solution, which behaves like a current wave launched at the unique edge of the array and decaying inside the structure.

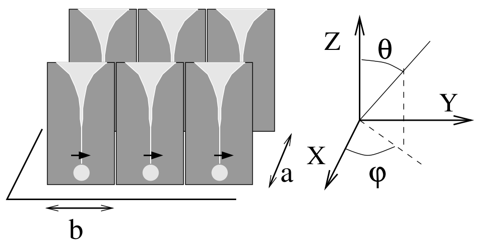

Figure 1: Reference configuration for the antenna array.

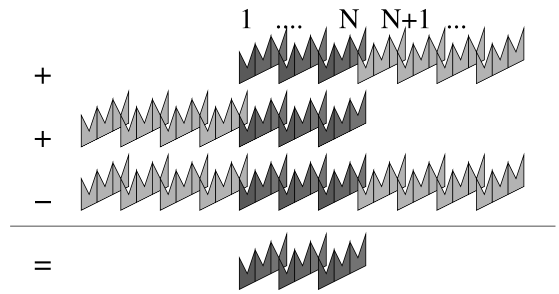

Figure 2: N-by-∞ array obtained using left and right APSIA's and the infinite array.

3 Solution of the APSIA problem

The equation system associated with the APSIA problem for $N = 3$ reads:

$$\begin{bmatrix} \mathbf{Z_0} & \mathbf{Z_1} & \mathbf{Z_2} \\ \mathbf{Z_{-1}} & \mathbf{Z_0} & \mathbf{Z_1} \\ \mathbf{Z_{-2}} & \mathbf{Z_{-1}} & \mathbf{Z_0} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \mathbf{x_3} \end{bmatrix} = \begin{bmatrix} \mathbf{v_1} \\ \mathbf{v_2} \\ \mathbf{v_3} \end{bmatrix} + \begin{bmatrix} \mathbf{v_1^s} \\ \mathbf{v_2^s} \\ \mathbf{v_3^s} \end{bmatrix} \tag{1}$$

where the $\mathbf{x_i}$ vectors correspond to the unknown current coefficients on successive antennas, and the $\mathbf{v_i}$ vectors are the excitation vectors associated with the different antennas. The $\mathbf{Z_p}$ MoM impedance sub-matrix is related to basis and testing functions located p array spacings apart in the finite-array direction. The $\mathbf{v_i^s}$ vectors correspond to the fields radiated by antennas $N+1$ to ∞, convoluted with the testing functions of antenna i:

$$\mathbf{v_i^s} = -\,\mathbf{Z_i^s}\ \mathbf{x}^\infty\ e^{-j\,n\,\psi_x} \tag{2}$$

where $\mathbf{Z_i^s}$ is computed in the same way as $\mathbf{Z^p}$, except for the Green's function, which corresponds to the Green's function related to a semi-infinite array. An efficient algorithm for the latter is described in [3]. The $\mathbf{Z_i^s}$ matrix only needs to be computed explicitely for $i = N$, for smaller values of i, the following recursion can be used:

$$\mathbf{Z_{i-1}^s} = (\mathbf{Z_i^s} - \mathbf{Z_{n-i+1}})\ e^{j\,\psi_x} \tag{3}$$

The solution procedure for a given APSIA problem is as follows:

- The left and right APSIA problems are solved in the brute-force way for N=3.
- The system of equations for the $N+1$ APSIA problem is established. This involves (*i*) the extension of the left member of equation (1), through the computation of $\mathbf{Z_N}$, and (*ii*) the update of the $\mathbf{v^s}$ vectors, which is obtained with the help of the recursion relation (3).
- The new equation system is solved iteratively, taking the $\mathbf{x_i}$'s obtained previously on antennas 0 to N, and the infinite-array solution on antenna $N+1$. The efficiency of the method results from the good quality of this first guess.

When the array is very large, the approach presented above is accelerated by exploiting the fact that the deviations w.r.t. the infinite-array solution can be described as waves scattered by the edges. A first consequence of this is that the solution of APSIA problems with increasing parameter N are very close to each other. Indeed, the difference between the true semi-infinite array and the APSIA resembles a wave propagating towards the inside the array, and therefore has a limited effect towards the edge. Hence, for large values of N, the currents on the first antennas do not need to be updated anymore. This would not be true if two edges were considered.

Besides this, when the finite-array dimension becomes very large, the edge-scattered wave can be extrapolated from the solutions obtained for the APSIA problems for smaller values of parameter N (say $N = n$). Denoting by $c(i)$ the port current at antenna i, the edge-scattered wave for $i > n$ is modeled as:

$$c(i) = c(n)\ (i/n)^{-q}\ e^{-j\,k\,(i-n)\,a} \tag{4}$$

where a is the element spacing, k is the free-space wavenumber and q is the damping exponent. Attempts have been made to obtain the exponent q from the solutions related to APSIA problems of smaller sizes. However, we observed that a slight underestimation could have dramatic effects on the gobal solution of the finite-by-infinite array. Hence, we decided to consider a fixed upper-bound value of $q = 1.5$.

4 Simulation results

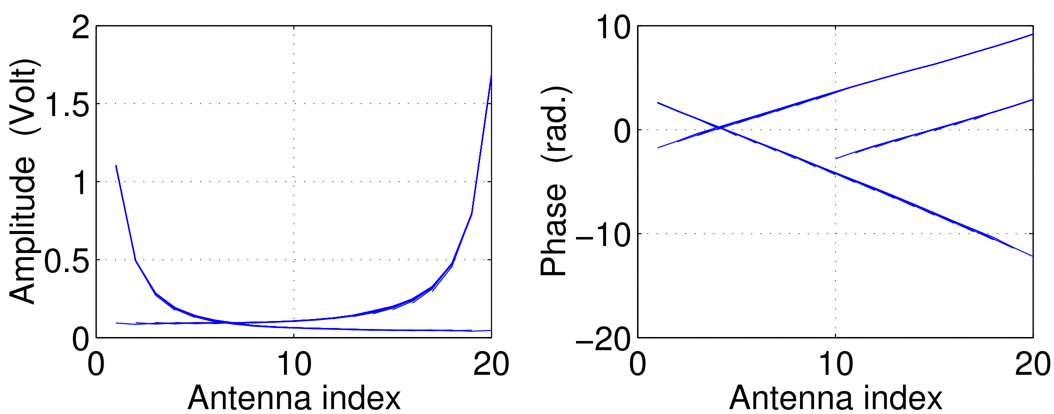

Figure 3: For an array infinite along $\hat{y}$, with 4 to 20 elements along $\hat{x}$, illuminated by a plane wave incident from $\theta = 30°$ and $\phi = 60°$, amplitudes and phases of the waves scattered by the edges. Spacings: 12.7 cm, wavelength: 100 cm.

Examples are shown for an array whose elements have a height of 23.8 cm and 12.7 cm spacings. The simulations are performed in receiving conditions, and yield the voltages impressed on the 100 Ω loads attached to the antennas. The antennas have 4 to 20 elements in the $\hat{x}$ direction and are infinite in the $\hat{y}$ direction. Fig. 3 shows the amplitudes and phases obtained for the left and right edge-scattered waves, obtained from the solutions of the two APSIA problems, for arrays of increasing sizes. Striking is that the phases are very close to linear, and that the amplitudes present very little change for APSIA problems of increasing dimensions. With the procedure described above, when solving for 20 antennas in the finite-array direction, the solutions for smaller arrays are implicitly generated. The intermediate results are shown in Fig. 4 (a vertical shift between solutions has been introduced for clarity).

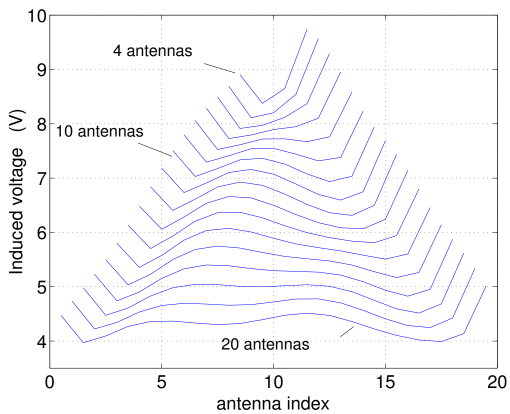

Figure 4: For the same array as above. Amplitudes of the impressed voltages for arrays of increasing sizes. Successive lines are shifted for clarity.

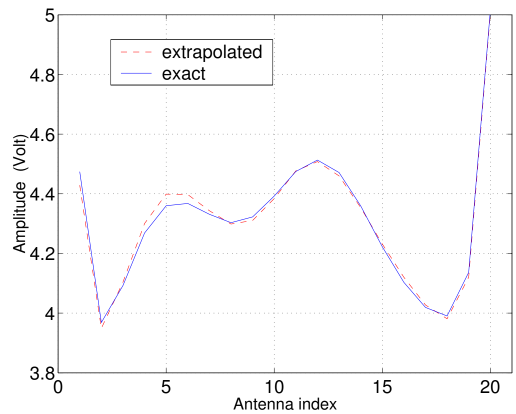

Figure 5: Exact solution for the 20-elements array, and extrapolated solution based on computations done for n=10.

Finally, Fig. 5 shows the solution extrapolated for 20 elements in the finite-array direction when the APSIA problems are computed for $n = 10$ elements. The match with the exact solution (solid line) is good. The largest deviation appears near element $n = 5$; it is due to the fact that, at the very low frequency considered here, a slight error on the extrapolation of the wave scattered by the right edge can have a non-negligible effect on the estimation of the currents on the left edge of the array (see Fig. 3, left).

5 Conclusion

The simulation of finite-by-infinite arrays provides insight into the strong edge effects characterizing broadband phased arrays. The waves scattered by the two edges can be obtained by decomposing the array with the help of two approximate semi-infinite array (APSIA) problems. The fact of computing the edge-scattered wave along its direction of propagation allows a rapid convergence from the order-n APSIA solution to the order $n + 1$ APSIA solution. All the solutions for arrays of intermediate sizes are obtained as a side product. For very large arrays, this procedure can be followed exactly, untill the difference w.r.t. the infinite-array solution is small enough, after which the edge-scattered wave is simply extrapolated.

References

[1] C. Craeye, "Decomposition of a finite-by-infinite array using two approximate semi-infinite arrays," *Journées Int. de Nice sur les Antennes*, vol. 2, pp., Nov. 2002.

[2] A. Neto, S. Maci, G. Vecchi, and M. Sabbadini, "A truncated Floquet wave diffraction method for the full wave analysis of large phased arrays – Part I: basic principles and 2-D cases," *IEEE Trans. Antennas Propagat.*, vol. 48, pp. 594-600, March 2000.

[3] C. Craeye, D.H. Schaubert, and A.G. Tijhuis, "An efficient computation scheme for the free-space Green's function of a two-dimensional semi-infinite phased array," *IEEE Trans. Antennas Propagat.*, in press.