A Multi-Cell Array Decomposition Approach to Composite Finite Array Analysis

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1. Introduction

Many real-world array systems can be decomposed into identical repeating cells, and in some cases multiple sets of like cells. Take for example a flat rectangular plate that can be decomposed into a planar grid of smaller plates. In much the same way, a finite array comprised of an arbitrary repeating cell, array decomposition can be used to greatly simplify and accelerate the storage and solution process [1]. Here this decomposition approach is extended to multiple cell structures for treating finite arrays consisting of multiple groups of like elements, such as finite arrays with an extended ground plane and radome of arbitrary periodicity. While the decomposition approach cannot be applied to a completely general structure devoid of repeating features, often some portion of a structure can benefit from array decomposition. Likewise, though a generalized approach can always be applied to analyze a structure with geometrical redundancies, exploitation of known redundancies typically leads to reduced storage and solution time. Further, the portion of the problem that cannot be decomposed into repeating cells can itself be treated as a single-element array, or may be decomposed into many unique single element arrays. A popular approach to large problems analysis is the Fast Multipole Method (FMM) [2]. In essence, FMM decomposes an arbitrary problem into a regular grid. However, in cases where geometrical redundancies exist, use of FMM alone is typically far slower than explicit decomposition [3, 4] of the pre-existing repeating features. Further, it is possible to combine FMM with the array decomposition method (ADM) as a hybrid approach [3] to best handle the repeating and non-repeating portions of a composite problem.

In this paper, we introduce a multi-cell array decomposition method applicable to complex structures with repeating features. With this method, we demonstrate that some common real-world structures can be decomposed into repeating cells for significant storage reduction, enabling us to analyze the coupling between several configurations of dual-polarized tapered-slot antenna arrays supported by a ground-plane. Measurement comparisons shall be presented. For brevity, we refer to this analysis technique at the multi-cell array decomposition method (multi-cell ADM).

2. Multi-Cell Decomposition Approach

For an introduction to ADM, the reader is referred to [1]. Much like the conventional ADM, the multi-cell ADM is carried out on *common* dimensions. In other words, for a single dimension, Z_{12} is the same as Z_{23} for sequential dimension numbering

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($Z_{12} = Z_{23} = Z_{(m-n)}$). Here, we will focus strictly on spatial or translational dimensions. A translational dimension is defined solely by a spacing parameter δr and a direction $\hat{r}$. By this definition, dimensions can be differentiated by a unique spacing between elements, or by aligning an array in a different spatial direction. In this way, complex systems can be constructed using multiple dimensions. For systems sharing a *common* dimension, a self-system Toeplitz property in addition to a cross-system Toeplitz property for element interactions can be exploited to decompose the system. An example of three systems sharing a *common* dimension is given in Figure 1. Each system consists of an independent number of elements, denoted M_α, but all three systems share a common δr and $\hat{r}$. We define systems 1, 2, and 3 as having unknowns n_1, n_2, and n_3, respectively. In a conventional approach to this problem, the total matrix storage requirements would be $O(M_1^2 n_1^2 M_2^2 n_2^2 M_3^2 n_3^2)$. This corresponds to the full system given in (1), though in general, the matrix layout would be different.

$$\begin{bmatrix}
\begin{bmatrix} [a_{11}]_{11} & \cdots & [a_{1M_1}]_{11} \\ \vdots & \ddots & \vdots \\ [a_{M_1 1}]_{11} & \cdots & [a_{M_1 M_1}]_{11} \end{bmatrix} &
\begin{bmatrix} [a_{11}]_{12} & \cdots & [a_{1M_2}]_{12} \\ \vdots & \ddots & \vdots \\ [a_{M_1 1}]_{12} & \cdots & [a_{M_1 M_2}]_{12} \end{bmatrix} &
\begin{bmatrix} [a_{11}]_{13} & \cdots & [a_{1M_3}]_{13} \\ \vdots & \ddots & \vdots \\ [a_{M_1 1}]_{13} & \cdots & [a_{M_1 M_3}]_{13} \end{bmatrix} \\
\begin{bmatrix} [a_{11}]_{21} & \cdots & [a_{1M_1}]_{21} \\ \vdots & \ddots & \vdots \\ [a_{M_2 1}]_{21} & \cdots & [a_{M_2 M_1}]_{21} \end{bmatrix} &
\begin{bmatrix} [a_{11}]_{22} & \cdots & [a_{1M_2}]_{22} \\ \vdots & \ddots & \vdots \\ [a_{M_2 1}]_{22} & \cdots & [a_{M_2 M_2}]_{22} \end{bmatrix} &
\begin{bmatrix} [a_{11}]_{23} & \cdots & [a_{1M_3}]_{23} \\ \vdots & \ddots & \vdots \\ [a_{M_2 1}]_{23} & \cdots & [a_{M_2 M_3}]_{23} \end{bmatrix} \\
\begin{bmatrix} [a_{11}]_{31} & \cdots & [a_{1M_1}]_{31} \\ \vdots & \ddots & \vdots \\ [a_{M_3 1}]_{31} & \cdots & [a_{M_3 M_1}]_{31} \end{bmatrix} &
\begin{bmatrix} [a_{11}]_{32} & \cdots & [a_{1M_2}]_{32} \\ \vdots & \ddots & \vdots \\ [a_{M_3 1}]_{32} & \cdots & [a_{M_3 M_2}]_{32} \end{bmatrix} &
\begin{bmatrix} [a_{11}]_{33} & \cdots & [a_{1M_3}]_{33} \\ \vdots & \ddots & \vdots \\ [a_{M_3 1}]_{33} & \cdots & [a_{M_3 M_3}]_{33} \end{bmatrix}
\end{bmatrix} \tag{1}$$

In (1), there are nine sub-systems, one for each intra-array coupling interaction, and one for each cross-coupling interaction between arrays. Because these three systems share a *common* dimension, the sub-systems are all non-symmetric Toeplitz (i.e. $Z_{12}^{\alpha\beta} = Z_{23}^{\alpha\beta} = Z_{(m-n)}^{\alpha\beta}$). Thus, each individual sub-system can be reduced from $O(M_\alpha M_\beta n_\alpha n_\beta)$ storage down to $O((M_\alpha + M_\beta - 1) n_\alpha n_\beta)$. This reduction corresponds roughly to the unshaded portion of the matrix in (1). Further, the contribution of each sub-system to the matrix-vector product operation of the iterative solution process can be accelerated with the equivalent of $n_\alpha n_\beta$ fast Fourier transforms (FFT) of length $M_\alpha + M_\beta - 1$.

Consider a composite finite array structure consisting of three unique cells (see Figure 2). The composite structure consists of a 6×5 horizontally polarized array, a 5×6 vertically polarized array, and ground plane, decomposed into a 7×7 grid. The assembled structure is a dual-polarized tapered-slot antenna in an egg-crate configuration. For this small configuration, the overall system storage can be reduced by 93% using the proposed multi-cell decomposition alone (larger systems have greater savings).

3. Example Simulation Results

Next we proceed to employ the proposed multi-cell ADM to analyze the 2^+ million unknown structure shown in Figure 3 (comparisons to measurements will be made for array-to-array coupling vs. scan angle at the presentation). This composite structure consists of two 10×11 and two 11×10 wideband tapered-slot antenna arrays, a ground plane, and surrounding shields. This composite structure requires a total of 13 multi-cells, though alternative decompositions can be made using multiple dimensions. Further, reuse of system resources can be accomplished by simply noting that many elements can be reconstructed by rotating cells. A straight-forward decomposition approach with 13 multi-cells reduces the problem size down to 14GB, compared to an unmanageable 16 terabytes without decomposition (three orders of magnitude difference). The electrically large wideband element alone takes 82MB to model using 2400 FEM, 2600 BI unknowns ($\sim 0.13 \times 1.4 \times 10$ inches, 2-4GHz).

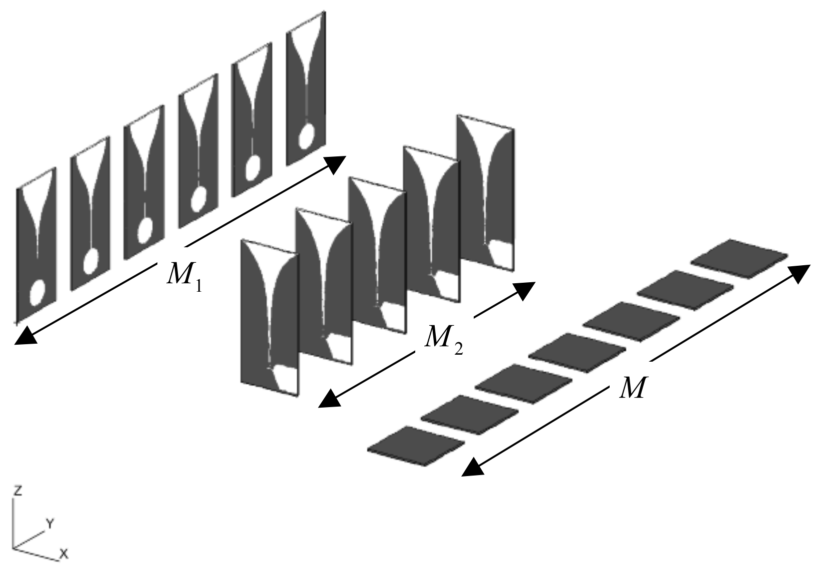

Figure 1. Illustration of dimensional decomposition.

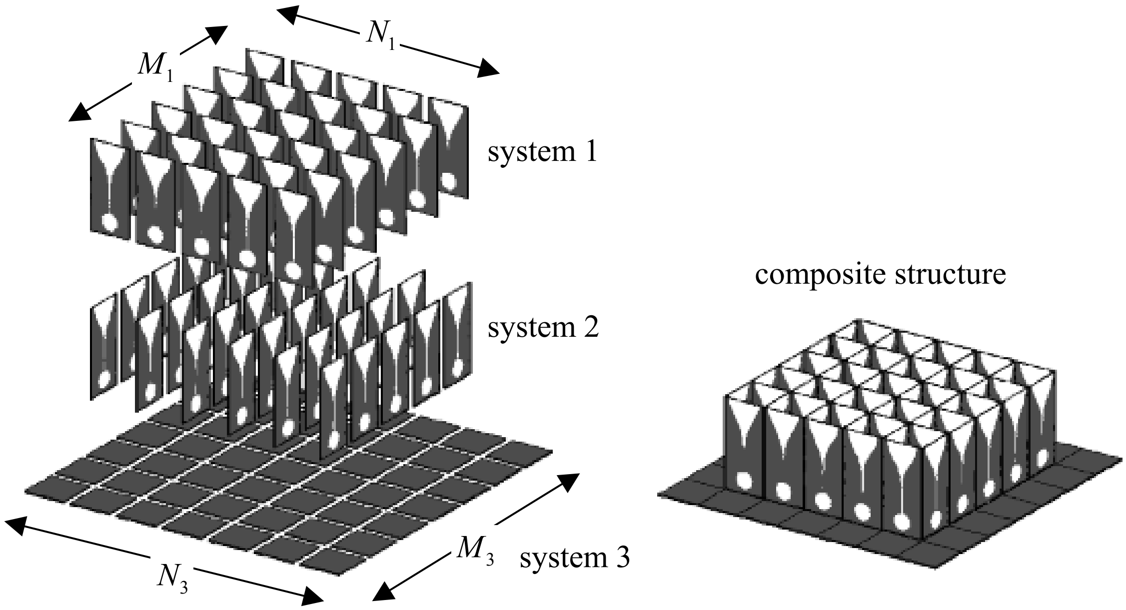

Figure 2. Decomposition of composite finite array structure.

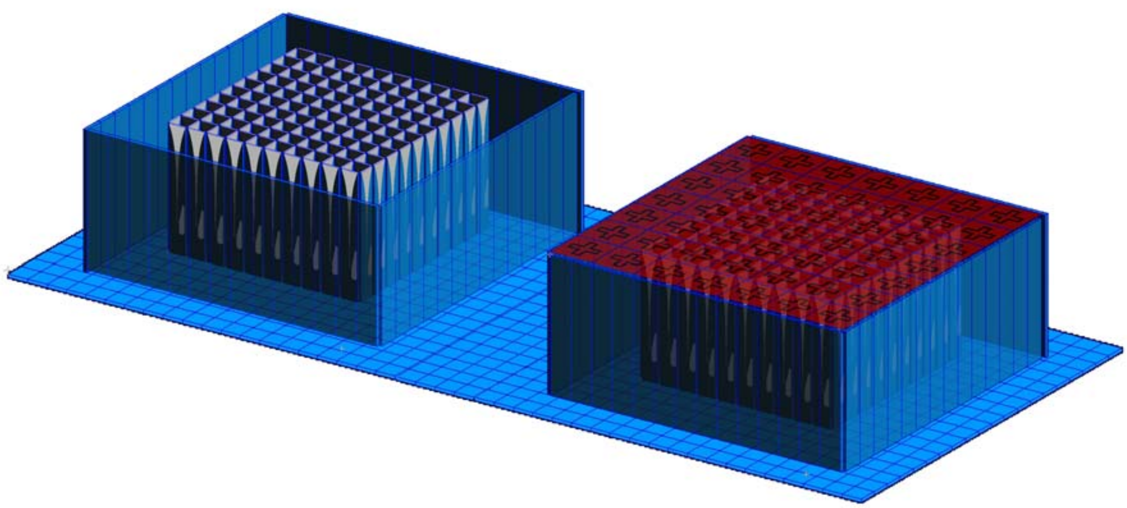

Figure 3. Array coupling test geometry.

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