A Fast Hybrid DFT-MoM for the Analysis of Large Finite Periodic Antenna Arrays in Grounded Layered Media

*P. Janpugdee[1], P. H. Pathak[1], P. Mahachoklertwattana[1], and P. Nepa[2]

[1]ElectroScience Lab, The Ohio State University, Columbus, OH 43212, USA
E-mail: pathak.2@osu.edu
[2]University of Pisa, Dept. of Information Engineering, Via Diotisalvi, 2-5612 Pisa, Italy

1 Introduction

The electromagnetic problem of the radiation and scattering from large finite arrays in grounded layered media is of interest in various applications. An integral equation technique based on the method of moments (MoM) is one suitable numerical approach for solving this type of problem. However, when the array size becomes very large, the conventional MoM can become highly inefficient or even intractable due to the computational cost and memory storage requirements.

A number of techniques and algorithms have been developed in the literature to reduce the computational time and also to overcome the memory storage limitations for solving large problems. In this work, a hybrid DFT-MoM approach has been developed to reduce the number of unknowns in the treatment of large finite arrays by using the Discrete Fourier Transform (DFT) for representing the global array distribution. Due to the compactness of the DFT spectrum of global array current distribution for most typical array excitations, the number of unknowns can be reduced from $\mathcal{O}(N \times M)$ to $\mathcal{O}(N + M)$, for $N \times M$ element arrays; hence reducing the computational time and memory storage requirements for the analysis of large finite arrays.

2 Formulation

A conventional EFIE-based MoM solution for solving electromagnetic problems of finite periodic antenna arrays in a grounded layer medium can be written in the form of matrix equation as

$$[Z][I] = [V] \tag{1}$$

where $[Z]$ is the impedance matrix, $[V]$ is the excitation vector, and $[I]$ is the unknown coefficient vector for the local basis over the antenna elements. The set of the unknown coefficients $\{I_{nm}\}$ in the vector $[I]$ represents the global array current distribution.

In general, we may have multiple expansion modes for the local basis over each antenna element. One can represent each set of the unknown coefficients $\{I_{nm}\}$ for each particular expansion mode in terms of a two-dimensional (2-D) DFT expansion as

$$I_{nm} = \sum_{p=0}^{N-1} \sum_{q=0}^{M-1} \tilde{I}_{pq} w_p^n w_q^m \tag{2}$$

or in the matrix form

$$[I] = [B][\tilde{I}] \tag{3}$$

0-7803-7846-6/03/$17.00 ©2003 IEEE

where $w_p = e^{-j2\pi p/N}$ and $w_q = e^{-j2\pi q/M}$ are the DFT global basis functions. $[\tilde{I}]$ is the unknown DFT coefficient vector and $[B]$ represents the DFT transformation matrix.

For most realistic transmitting phased array antenna excitations as well as for external plane wave excitation, the DFT spectrum of the array current distribution is highly localized as illustrated in Figure 1. This local phenomenon of the spectral distribution can be explained via the UTD concepts for finite arrays [1]. As a result, only the dominant spectral components are significant and sufficient to represent the original global array current distribution. Thus, $\{I_{nm}\}$ can be approximated by a highly compact truncated 2-D DFT expansion

$$I_{nm} \simeq \sum_{p,q \in D} \tilde{I}_{pq} w_p^n w_q^m \tag{4}$$

or in the matrix form

$$[I] \simeq [B_t][\tilde{I}_t] \tag{5}$$

where $\{p, q \in D\}$ represents the indices of dominant DFT spectrum. $[\tilde{I}_t]$ and $[B_t]$ represent the truncated versions of $[\tilde{I}]$ and $[B]$, respectively. Consequently, the length of $[\tilde{I}_t]$ is much smaller than that of $[I]$ or $[\tilde{I}]$.

By using the approximation in (5), the MoM matrix equation in (1) can be replaced by

$$[Z][B_t][\tilde{I}_t] = [V] \tag{6}$$

Applying the conjugate Galerkin testing with the same set of global truncated DFT basis functions on (6), one obtains

$$[B_t]^\dagger[Z][B_t][\tilde{I}_t] = [B_t]^\dagger[V] \tag{7}$$

or

$$[\tilde{Z}_t][\tilde{I}_t] = [\tilde{V}_t] \tag{8}$$

where † denotes "conjugate transpose". The resulting matrix equation in (8) is relatively more compact than the original one in (1), thus substantially reducing the overall computational cost for a matrix solution.

3 Computational Improvement

In the present work, the elements of impedance matrix $[Z]$ are efficiently computed by employing the asymptotic closed form Green's function for a grounded multilayered medium [2] when the source and field point separations greater than one free space wavelength; while for smaller separations, the Green's function is computed numerically from the usual Sommerfeld-type integral representation. The asymptotic closed form Green's function allows a significantly faster computation of the impedance matrix elements than the highly inefficient and extremely time consuming computation of the numerical Sommerfeld-type integral representation for the field. Only the diagonal and near diagonal elements of $[Z]$ are really computed by the numerical integration whereas the others are computed by using the much more efficient asymptotic closed form result mentioned above. Furthermore, due to the periodic property of the array, the computation of the impedance matrix $[Z]$ results in only $\mathcal{O}(N \times M)$ of the required CPU time and memory storage for $N \times M$ element arrays.

In the present hybrid DFT-MoM approach, some small additional CPU time and memory storage is needed in the matrix transformation and compression process; i.e., to perform the operations $[B_t]^\dagger[Z][B_t]$ and $[B_t]^\dagger[V]$. The highly efficient standard Fast Fourier Transform (FFT) algorithm is employed to perform the 2-D DFT transformation in such process so that the operation $[B_t]^\dagger[Z][B_t]$ requires $\mathcal{O}(\alpha \times N \times M \times \max\{N, M\})$ CPU time and $\mathcal{O}(N \times M \times \max\{N, M\})$ memory storage, where α is a constant related to the number of prime factors of N and M. Typically, α is much less than N and M for moderate to large N and M. Also, the operation $[B_t]^\dagger[V]$ requires $\mathcal{O}(\alpha \times N \times M)$ CPU time and $\mathcal{O}(N \times M)$ memory storage. As a result, the net computational time in this process is much less than the time for constructing the impedance matrix and for solving the matrix equation.

The compressed matrix equation in (8) is $\mathcal{O}(N+M)$, which is much more compact than the original matrix equation of $\mathcal{O}(N \times M)$ in (1). Thus, it requires only $\mathcal{O}((N + M)^2)$ CPU time for solving by the iterative matrix solver, while the conventional approach requires $\mathcal{O}((N \times M)^2)$ CPU time. Typically, $N + M \ll N \times M$ for large to very large arrays; hence, the hybrid DFT-MoM significantly reduces the computational time for solving the matrix equation compared to the conventional MoM. The significant computational time reduction in this step completely overcomes the far less additional time required in the matrix transformation and compression process mentioned above. Furthermore, the computational time saving is much more significant while computing any quantities as a function of the scan angle, e.g. the Radar Cross Section (RCS), where it is required to solve the matrix equation for each scan angle as one steps through the whole range of scan angles.

4 Numerical Results

The numerical results in Figure 2 illustrate the accuracy and efficiency of the DFT-MoM comparing to the conventional MoM for the analysis of large finite arrays. The simulations are run on a $N \times M$ patch array on a single grounded dielectric layer. The grounded layer has a dielectric constant 12.8, a thickness $0.06\lambda_0$, and is assumed to extend to infinity while the number of array elements is finite. Each microstrip patch has a dimension of $L \times W = 0.1074\lambda_0 \times 0.15\lambda_0$, and is fed at the center of one edge by microstrip feed line.

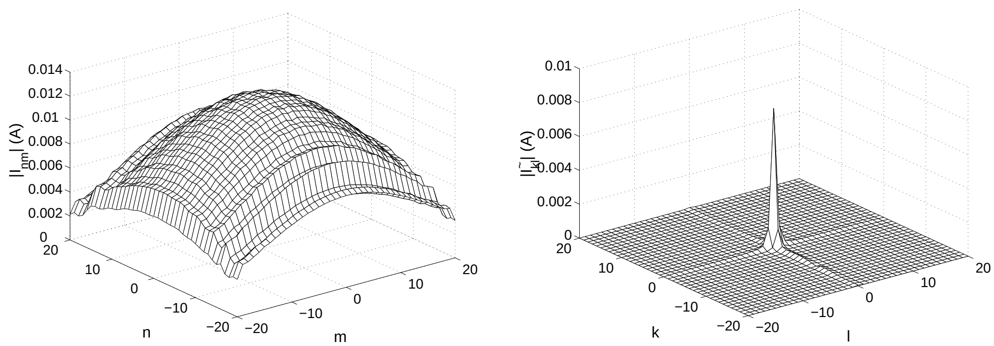
Figure 2(a) shows the radiation efficiency of 19×19 array for various scan angles. The plot shows the scan blindness of a finite array which occurs in this geometry at the scan angle around 46^o. The DFT-MoM solution agrees very well with the conventional MoM solution while the computational time saving factor is about 1:50 or better for this case. For larger arrays, the time saving is expected to significantly more. This result also compares extremely well with that given by [3]. Figure 2(b) shows the CPU time required by the DFT-MoM and the conventional MoM as a function of the array size, where it is noted that the impedance matrix $[Z]$ is computed via the asymptotic closed form solution for off-diagonal elements in both the DFT-MoM as well as the conventional MoM. The significant computational time saving by the DFT-MoM can be seen from the plot.

References

[1] P. Janpugdee, *An Efficient Discrete Fourier Transform Based Ray Analysis of Large Finite Planar Phased Arrays*, Master's thesis, The Ohio State University, 2002.

[2] C. W. Chuang and P. H. Pathak, *Moment Method Solutions of Phased Arrays of Printed Cross-Dipoles in a Grounded Multilayer Dielectric Medium*, Unpublished notes, The Ohio State University, 2002.

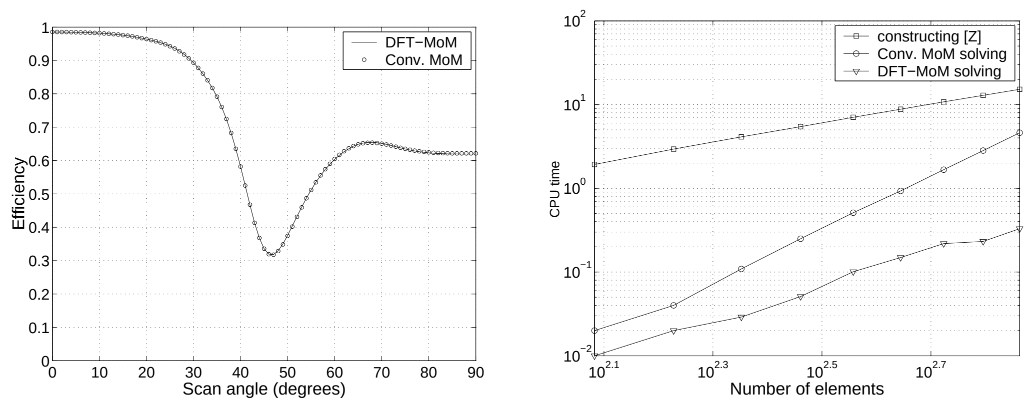
[3] D. M. Pozar, "Finite Phased Arrays of Rectangular Microstrip Patches," *IEEE Trans. Antennas Propagat.*, vol. AP-34, no. 5, pp. 658-665, May 1986.



(a) Global array current distribution

(b) DFT spectrum

Figure 1: A typical example of (a) the global array current distribution and (b) the corresponding DFT spectrum, for a tapered array excitation.



(a) The radiation efficiency of a 19×19 array

(b) The computational time

Figure 2: The comparisons of (a) accuracy and (b) efficiency of the hybrid DFT-MoM solution to the conventional MoM solution.