

## Dyadic Eigenfunctions and Natural Modes for Hybrid Waves in Planar Media

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The typical analysis of waveguiding problems in electromagnetics involves the formulation of differential equations. Eigenvalues and eigenfunctions of the resulting differential operators are sometimes studied. More often, one considers the natural modes of the structure, which are the homogeneous solutions of the differential equation, and can be considered as eigenfunctions corresponding to zero eigenvalues (at least in a generalized sense).

For planarly-layered media, it is well-known that TE, TM, and hybrid modes may exist, depending on the excitation. The usual analysis consists of analyzing the TE and TM natural modes by reduction to scalar equations, and considering these (uncoupled) solutions as the fundamental modes of the structure. If a source excites a hybrid mode, it is constructed out of a linear combination of the fundamental TE and TM modes.

In this paper an alternative method is developed. The governing eigenvalue equations are elevated to the dyadic level, leading to dyadic eigenfunctions and dyadic natural modes. In particular, the dyadic natural modes are generally hybrid, and, as special cases, decouple to result in the familiar TE and TM independent modes. In the general case, the natural relationship between hybrid mode components is obtained via the corresponding relationship between scalar components of the dyadic natural mode. Dyadic adjoint modes and dyadic associated functions are also discussed, and orthogonality properties of the dyadic modes are obtained.