# Generalization of a Fundamental Theorem for the Scattering from a Receiving Antenna

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The scattering properties of a receiving antenna is important for the determination of the radar scattering cross section of a receiving antenna, and also for the evaluation of local interference effects.

An old fundamental theorem, derived more than half a century ago, states that the electric field $\mathbf{E}_s(Z_L)$ scattered by a receiving antenna terminated in a load impedance $Z_L$ is given by

$$\mathbf{E}_s(Z_L) = \mathbf{E}_s(Z_L = 0) + \frac{Z_L}{Z_L + Z_A} I_{sc} \mathbf{E}_r \tag{1}$$

where $\mathbf{E}_s(Z_L=0)$ is the field scattered when the receiving antenna is terminated in a short circuit, $Z_A$ is the antenna input impedance when it is transmitting, $I_{sc}$ is the current that flows into the shorted termination when it is receiving (Norton equivalent circuit current source), and $\mathbf{E}_r$ is the field the receiving antenna radiates when it is transmitting with unit current at its input. Aharoni (J. Aharoni, "Antennas", Clarendon Press, Oxford, 1946, pp.164-176) calls the second term on the right hand side in (1) the re-radiated field. In this paper we derive a generalized form of this scattering formula which is given by

$$\mathbf{E}_s(Z_L) = \mathbf{E}_s(Z_r) + I(Z_L = Z_r) \frac{Z_L - Z_r}{Z_L + Z_A} \mathbf{E}_r \tag{2}$$

In this formula $\mathbf{E}_s(Z_r)$ is the field scattered by the receiving antenna when it is terminated in the reference impedance $Z_r$ , and $I(Z_L=Z_r)$ is the current flowing into $Z_r$ . When $Z_r=0$ (2) reduces to (1). A case of special interest is when the antenna is connected to a transmission line with characteristic impedance $Z_c$ and the reference impedance is chosen equal to $Z_c$. For this case (2) becomes

$$\mathbf{E}_s(Z_L) = \mathbf{E}_s(Z_c) + I(Z_L = Z_c) \frac{Z_L - Z_c}{Z_L + Z_A} \mathbf{E}_r \tag{3}$$

If now $Z_L=Z_c$ the re-radiated portion of the scattered field vanishes. There is no power flow in the transmission line towards the antenna input so physically it is easy to understand that there will be no re-radiated field. We propose to call the residual scattered field $\mathbf{E}_s(Z_c)$ the intrinsic scattered field. In general the far zone radiation pattern of the intrinsic scattered field is not the same as the radiation pattern of the receiving antenna when it is used to transmit. The generalized scattering formula (2) was derived using the fundamental principles of uniqueness and superposition for electromagnetic fields.