A new FDTD model for lightning generated EM wave simulation

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1 Abstract

A new full-wave finite-difference time domain method (FDTD) model was developed to simulate lightning-generated electromagnetic wave propagation in the ionosphere. A new PML technique (NPML) is applied in this model, which performs better than the classic PML in this application. A numerical method is used to derive the coefficients of FDTD difference equations. Dispersion relations are analyzed to investigate the error of the model and the stability conditions are discussed. Simulation results show the improvement of this new model compared with a previous version.

2 Introduction

The electromagnetic wave radiated by intense lightning discharges affects the atmosphere through different processes. To explore the mechanisms of these processes quantatively, a complete evaluation of lightning-generated electromagnetic fields at different locations is necessary. Previous work addressing the same issue is based on the Earth-ionosphere waveguide theory [1], [2] or makes some assumptions [2], which leads to some limits. Without those assumptions and treating the ionosphere as a true cold plasma, this numerical model is a fully electromagnetic 2-D cylindrical model developed by using FDTD combined with a new PML technique. For efficiency, we derive the coefficients of the difference equations of FDTD numerically. Being valid throughout the ionosphere, this model is a powerful tool for the study of lightning-related phenomena. It can be used to select the structure and orientation of antennas in space measurements and effective measurement methods at different altitudes.

3 Governing Equations

The ionosphere can be regarded as cold plasma with the Earth's magnetic field superposed as long as the energy generated by lightning is not high enough to modify the medium. The three important properties of a cold plasma which are used in this model are plasma frequency ω_{pn}, gyrofrequency ω_{Bn} and collision frequency ν_n of different charged species denoted by n. The fields in this medium are described by Maxwell's equations,

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J_{tot}} \tag{2}$$

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coupled to an equation for the electric current [3]

$$\frac{\partial \mathbf{J_n}}{\partial t} + V_n \mathbf{J_n} = \frac{\mathbf{q_n}}{|\mathbf{q_n}|} \omega_{\mathbf{Bn}} (\mathbf{J_n} \times \mathbf{b_E}) + \epsilon \omega_{\mathbf{pn}}^{\mathbf{2}} \mathbf{E} \tag{3}$$

where $\mathbf{b_E}$ is defined as the unit vector in the direction of the Earth's magnetic field.

4 Technique details, numerical results and discussion

4.1 Finite difference algorithm and discretization schemes

Because FDTD method can easily simulate transient EM wave propagation in an inhomogeneous, anisotropic medium, we choose FDTD to develop this model. Two FDTD methods are considered here referred to as H-J collocated method [4] and E-J collocated method [5]. Both of these methods are second order accurate. However, the stability condition of E-J method is remained same as the free space case while the stability condition of the H-J method depends on the parameters of the plasma. For 1-D problems without the Earth's magnetic field, the maximum Courant numbers for H-J method and E-J method are $\sqrt{1-(\omega_{pn}\Delta t/2)^2}$ and 1, respectively. Therefore, for efficiency, we choose E-J method here.

4.2 Numerical derived coefficient method

It is tedious to derive explicit difference equation coefficients analytically. A new method is used to derive the coefficients of FDTD difference equations to accelerate the coding and simulation. The difference equations can be written implicitly as:

$$A_1 X_1^n = B_1 X_1^{n-1} + C_1 Y_1^{n-1/2} \tag{4}$$

where X_1^n is the field value vector to be updated and A_1, B_1, C_1 are coefficient matrixes. Then

$$X_1^n = (A_1^{-1} B_1) X_1^{n-1} + (A_1^{-1} C_1) Y_1^{n-1/2} \tag{5}$$

and the explicit coefficients can thus be found numerically, which is convenient for coding and can save computation costs. One point must be mentioned for this method is that the field values have to be scaled to avoid ill-conditional problems when solving this matrix equation.

4.3 New PML technique

A new PML technique, the NPML [6] is used in this model. The classic PML technique [7] works well in many applications. However, in this particular application with the presence of dispersive, lossy, and anisotropic medium, the NPML is easier to implement than the classic PML. Furthermore, by using this new PML technique, some computation costs can be saved.

4.4 Dispersion relation and error analysis

To evaluate the accuracy of this numerical method, dispersion analysis is an appropriate technique. The difference between the analytical dispersion relation and the numerical dispersion relation reflects the accuracy of the numerical method.

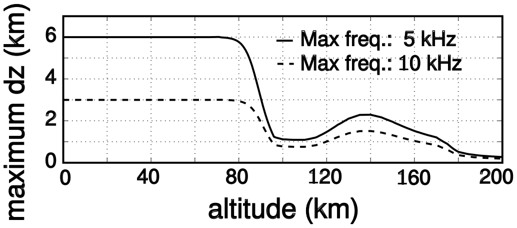

Figure 1: Derived maximum FDTD cell size dz profile versus altitude

In cold plasma with a static magnetic field, two propagation modes exist. With the aid of the dispersion relation analysis, given the ionosphere profiles, error tolerance, propagation direction and frequency band, we can derive a profile of maximum dz (cell size) versus altitude. Here we consider the case of a vertically propagating wave. The analysis results are shown in Figure 1 (for accuracy purpose, assuming the maximum dz is 1/10 of the wavelength).

Based on the dispersion analysis results, if we want use dz as 1 km, then we can ensure the accuracy up to 5 kHz if the maximum altitude does not exceed 175 km. By using this scheme, the resulting relative dispersion and dissipation errors for both modes are shown as solid curves in Figure 2. The relative dispersion error is within 2% and the relative dissipation error is controlled within 0.2%.

4.5 Comparison with previous model

The previous model [2] makes an assumption that $\nu_n >> \omega$. However, this assumption is not valid at high altitude where the collision frequencies of particles are quite low.

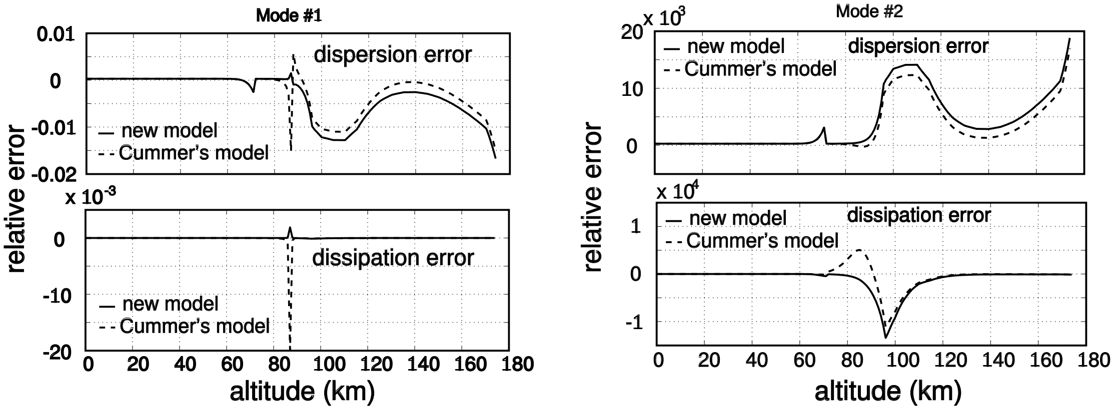

Figure 2: Relative dispersion and dissipation error for the new model and previous model

Also consider the same case as the previous subsection.where dz=1 km and frequency is 5 kHz. As the results shown in Figure 2, the dissipation error of mode ♯ 1 for the previous model has much bigger error than the new model around 86 km above the ground. Obviously, this assumption introduces much extra energy loss at high altitudes.

Figure 3 shows the simulation results (E_ϕ) at 150 km above the ground generated by a lightning current with a Gaussian waveform. The (E_ϕ) component at high altitude calculated by the previous model is much weaker than the new model. We conclude that the new model is more accurate than the old model, especially at high altitudes.

5 Conclusions

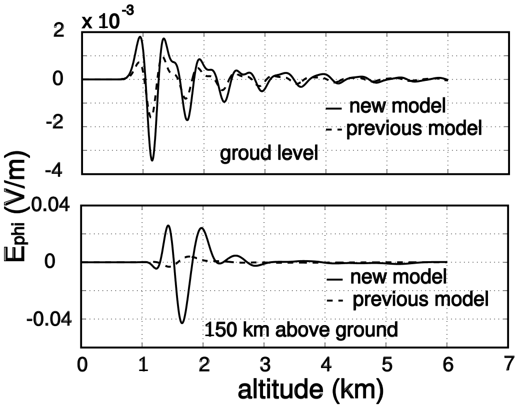

Figure 3: Comparison between the new model and the previous model

This new FDTD model for lightning-generated EM wave simulation treats the ionosphere as a true cold plasma. After comparing the advantages and disadvantages of H-J collocated method and E-J collocated method, we find E-J method is more preferable in consideration of simulation time. After analyzing dispersion relations for both propagation modes, we find the accuracy requirements can be met by carefully choosing cell size and the maximum altitude of the computation domain. Also we find the assumption made in the previous model does not affect the dispersion phenomenon too much. However, it does introduce extra energy loss, which results in a huge error in amplitudes at high altitudes. The numerical method for deriving the coefficients of FDTD difference equations simplifies the tedious analytical computation. The NPML is easier to apply to this problem than the classic PML and it saves some computation costs. Future work will focus on extending the frequency band and making the simulation running more efficiently by applying multi-grid FDTD or high order FDTD to the model.

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