Modal Propagation in a Circular Waveguide with a Rough Wall

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In order to develop an improved understanding of electromagnetic wave propagation in possibly rough-walled regions such as caves and tunnels, we first consider the effects of a randomly rough wall on the propagation of electromagnetic modes in a perfectly conducting quasi-circular waveguide. We wish to determine the utility of a modal field representation, investigate the partitioning of mode power between coherent and incoherent parts of the field, and describe the statistical properties of the modal fields in terms of the statistical properties of the wall roughness.

The field is represented in terms of two scalar functions that satisfy the scalar Helmholtz equation. Forcing the tangential electric field to vanish on the rough wall, we make use of the Wiener-Ito stochastic functional calculus to represent the resulting conditions on the unknown functions that describe the field. We show that the problem becomes one of solving an infinite set of coupled stochastic integral equations for these functions. We develop an approximate solution to the coupled integral equations by using an iteration scheme that is initialized to represent a mode in an equivalent smooth-walled guide. The radius of this smooth-walled guide is less than the average radius of the rough-walled guide, the reduction depending on the variance of the wall roughness, and the modal cutoff frequency is correspondingly higher.

It is shown that this process yields a representation for the modal field that comprises a coherent part similar to the modal field in an equivalent smooth-walled guide of reduced radius, and an incoherent part whose expected value is zero and whose variance depends on radial position within the guide. The variance is largest near the rough wall and decreases as the observation point moves toward the axis of the guide. Representative numerical results are presented to illustrate the analysis; and inferences are drawn regarding the problem of wireless communication in tunnels with rough walls.