Efficient Semi-Analytical Analysis of Two-Dimensional Photonic Crystals

Davy Pissoort*, Daniël De Zutter and Frank Olyslager.
Department of Information Technology (INTEC), Ghent University
St.-Pietersnieuwstraat 41, B-9000 Gent, Belgium
Tel: 3292643343 Fax: 32932643593
E-mail: davy.pissoort@intec.rug.ac.be

Abstract— In this paper, the fields in a 2D finite photonic crystal are calculated in a rigorous and efficient way using an integral equation technique. At the surface of every cylinder an approriate boundary impedance is used. The validity and accuracy of the method are verified by comparing the numerical results for a small example with those obtained by the method of Auxiliary Sources. Finally, this method is used to study a more complex PBG structure, namely a wave demultiplexer.

I. Introduction

Photonic crystals (PCs) have raised substantial interest because of their ability to control the light-wave propagation [1]. These PCs consist of a set of parallel cylinders embedded in a homogeneous medium or planar stratified medium. Because of the periodicity, a PC exhibits photonic band gaps in which electromagnetic fields cannot propagate in given directions. By removing a row of cylinders, channel waveguides, operating at frequencies within the bandgap, are created. These waveguides are expected to provide waveguiding with low losses and to allow sharp bends. The finite-difference time-domain (FDTD) method has been widely used to simulate the properties of these PCs. With respect to frequency domain tecniques such as integral equations, it is often stated that they are very time consuming. In this paper however, we present a rigorous and very efficient semi-analytical technique to calculate the fields in a 2D finite photonic crystal. In this contribution we restrict ourselves to a homogeneous background medium and to TE polarization.

II. Basic Formulation

A. Field and current representation

At the surface of every cylinder i (Fig. 1) we define unknown surface currents

$$J_z^i(\rho_i = a_i, \phi_i) = \sum_{m=-\infty}^{+\infty} \frac{I_m^i}{2\pi a_i} e^{jm\phi_i}, \tag{1}$$

with a_i the radius of cylinder i and (ρ_i, ϕ_i) the local cylindrical coordinates of this cylinder. These currents are placed in an infinite homogeneous region with dielectric permittivity ϵ_{ext}. The total electric field on the surface of cylinder i can also be written as a Fourier series

$$E_z^i(\rho_i = a_i, \phi_i) = \sum_{p=-\infty}^{+\infty} e_p^i e^{jp\phi_i}, \tag{2}$$

with

$$e_p^i = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_z^i(\rho_i = a_i, \phi_i) e^{-jp\phi_i} d\phi_i. \tag{3}$$

The total field $E_z^i(\rho_i = a_i, \phi_i)$ is the superposition of the following 3 contributions:
(i) the field caused by the surface current on cylinder i: $E_z^{ii} = \sum_{p=-\infty}^{+\infty} e_p^{ii} e^{jp\phi_i}$;
(ii) the field caused by the surface currents on the other cylinders: $E_z^{ji} = \sum_{p=-\infty}^{+\infty} \sum_{j\neq i} e_p^{ji} e^{jp\phi_i}$;

(iii) the field caused by the source: $E_z^{0i} = \sum_{p=-\infty}^{+\infty} e_p^{0i} e^{jp\phi_i}$.
One can easily verify that the self-patch contribution is given by

$$e_p^{ii} = -(\frac{\omega\mu_0}{4})H_p^{(2)}(ka_i)J_p(ka_i)I_p^i. \tag{4}$$

In order to calculate the non self-patch contribution e_p^{ji}, we express the field caused by the currents on cylinder j in terms of the local coordinates of cylinder i using the addition theorem of the Hankel functions. One finds

$$e_p^{ji} = -\frac{\omega\mu_0}{4} \sum_{m=-\infty}^{+\infty} H_{p-m}^{(2)}(kR_{ij})J_m(ka_j)J_p(ka_i)e^{j(m-p)\Phi_{ij}} I_m^j, \tag{5}$$

with R_{ij} and Φ_{ij} respectively the distance and the angle between the centers of both cylinders, see Fig. 1.

B. *Boundary Impedance*

For the case of perfectly conducting cylinders ($epsilon_i = \infty$), the current (1) is the physical surface current and the whole problem can be solved by demanding that the total electric field is zero on the surface of all cylinders. This in turn leads to a set of equations for the unknown surface currents I_m^i in (1). For dielectric cylinders, internal fields have to be taken into account. Here, we circumvent this problem by still using the surface current (1) as our only unknown quantity, but now an appropriate boundary impedance has to be introduced relating the total external electric field to this current. For the calculation of this boundary impedance, we consider two situations (Fig. 2):

- the original situation with a dielectric cylinder with radius a_i and dielectric permittivity ϵ_i, placed in a homogeneous space with relative dielectric permittivity ϵ_{ext};
- a homogeneous space with relative dielectric permittivity ϵ_{ext}, but with unknown currents $J_z^i = \sum_{n=-\infty}^{+\infty} I_n^i e^{jn\phi_i}$ placed on the now fictitious surface S_i of the cylinder i.

In the first situation the fields inside the dielectric cylinder are given by

$$E_z^I(\rho_i \leq a_i, \phi_i) = \sum_{n=-\infty}^{+\infty} A_n J_n(k_0\sqrt{\epsilon_i}\rho_i)e^{jn\phi_i}, \tag{6}$$

$$j\omega\mu_0 H_{\phi_i}^I(\rho_i \leq a_i, \phi_i) = k_0\sqrt{\epsilon_i} \sum_{n=-\infty}^{+\infty} A_n J_n'(k_0\sqrt{\epsilon_i}\rho_i)e^{jn\phi_i}, \tag{7}$$

where A_n are unknown coefficients. In the second situation the fields at $\rho_i = a_i^+$ are given by

$$E_z^{II}(a_i^+, \phi_i) = \sum_{n=-\infty}^{+\infty} B_n J_n(k_0\sqrt{\epsilon_{ext}}a_i)e^{jn\phi_i}, \tag{8}$$

$$j\omega\mu_0 H_{\phi_i}^{II}(a_i^+, \phi_i) = k_0\sqrt{\epsilon_{ext}} \sum_{n=-\infty}^{+\infty} B_n J_n'(k_0\sqrt{\epsilon_i}a_i)e^{jn\phi_i} + j\omega\mu_0 \sum_{-\infty}^{+\infty} \frac{I_n^i}{2\pi a_i} e^{jn\phi_i}. \tag{9}$$

We define the boundary admittance $Y_n^i = \frac{1}{Z_n^i}$ as

$$\frac{I_n^i}{2\pi a_i} = Y_n^i e_n^{II}(a_i) = Y_n^i B_n J_n(k_0\sqrt{\epsilon_{ext}}a_i). \tag{10}$$

This boundary admittance can be determined by demanding that $E_z^I(a_i^-, \phi_i) = E_z^{II}(a_i^+, \phi_i)$ and that $H_{\phi_i}^I(a_i^-, \phi_i) = H_{\phi_i}^{II}(a_i^+, \phi_i)$:

$$Z_n^i = j\sqrt{\frac{\mu_0}{\epsilon_0}} \frac{J_n(k_0\sqrt{\epsilon_{ext}}a_i)J_n(k_0\sqrt{\epsilon_i}a_i)}{\sqrt{\epsilon_{ext}}J_{n+1}(k_0\sqrt{\epsilon_{ext}}a_i)J_n(k_0\sqrt{\epsilon_i}a_i) - \sqrt{\epsilon_i}J_{n+1}(k_0\sqrt{\epsilon_i}a_i)J_n(k_0\sqrt{\epsilon_{ext}}a_i)}. \tag{11}$$

A perfectly conducting cylinder corresponds with $\epsilon_i = \infty$, or $Z_n^i = 0$ as expected. The unknown surface currents are then determined by

$$E_z^i(a_i, \phi_i) = Z_n^i I_n^i \text{ or } E_z^{tot}(a_i, \phi_i) - \sum_{n=-\infty}^{+\infty} Z_n^i I_n^i e^{jn\phi_i} = 0. \tag{12}$$

In practice, this infinite sum has to be truncated to a finite number of terms: n going from $-N_m$ to $+N_m$. The main advantage of our approach is that the only unknowns of the complete problem are now the surface currents I_n^i. The fields inside the cylinders have been eliminated on beforehand.

C. Comparison with the method of Auxiliary Sources

In the method of Auxiliary Sources [2], one introduces two sets of N_c auxiliary sources for every cylinder. The first set is radiating in a homogeneous medium with permittivity ϵ_{ext} and is located on an auxiliary surface $\hat{S}_i$ located inside the surface S_i. With this set one calculates the scattered field outside the surface S_i. The second set is radiating inside an infinite space filled by the material of the dielectric cylinder i and is located at an auxiliary surface $\tilde{S}_i$ outside the surface S_i. This set describes the total field inside the cylinder i. The boundary conditions are imposed at a discrete set of N_p points on the surface S_i in order to determine the unknown currents. Fig. 3 shows the total electric field calculated with both methods for the geometry shown at the left of the figure. It consists of 5 cylinders placed in vacuum on a circle with radius $0.5a$ with a line source placed at its centre. The radii of the 5 cylinders are respectively $1.6a$, $1.8a$, $2a$, $1.7a$ and $2.5a$. They all have a relative dielectric constant of 8. The amplitude of the electric field is shown along the dashed circle with a radius of $0.5a$ for a frequency of $0.5\frac{c}{a}$. As can be seen, the required number of unknowns in our method (5 x 5 = 25) is much less then that in the method of AS (5 x 80 = 400) in order to reach the same accuracy.

III. Example: demultiplexer

A wave demultiplexer is a very important device for improving the capacity of optical communications. A photonic crystal can also be applied to construct such a device. In [3] a kind of wave demultiplexer realized through direct resonant tunneling between waveguides and points defects, was proposed. The schematic of a wave demultiplexer with two channels is shown in Fig. 4. At the end of the inlet waveguide, the waveguide bifurcates into two waveguides that are linked with a resonant cavity, each formed by five cylinders. These resonant cavities are different from each other. In this way, one obtains two different frequencies in the two outlets of the demultiplexer. The parameters of the device are as follows. The radius of the cylinders is $0.182a$, with a the lattice constant. The dielectric cylinders have a relative dielectric constant of 8.0 and are placed in a homogeneous medium with a relative permittivity of 1.03. In Fig. 4 two special cylinders $C1$ and $C2$ are placed in the resonant cavities. Their radius is $0.355a$ and their relative permittivities are respectively 6.5 and 4.5. In the simulations, a current line source is placed in the inlet waveguide. In total there are 509 dielectric cylinders in this example. For the first outlet the output frequency is $f = 0.406\frac{c}{a}$ and for the second outlet this is $f = 0.445\frac{c}{a}$, which corresponds well with the results given in [3]. The electric field at the outlet of the demultiplexer is shown at the right of Fig. 4 for these two frequencies and for the intermediate frequency $f = 0.425\frac{c}{a}$, at which there is almost no signal at the outlet of the demultiplexer. With $N_m = 2$, or 509 x 5 = 2545 unknowns, the calculation time for one frequency was about 3 minutes. Increasing the number of unknowns N_m causes only very minor changes, indicating that the method has converged.

IV. Acknowledgement

D. Pissoort is a Research Assistant of the Fund for Scientific Research - Flanders (Belgium)(F.W.O. - Vlaanderen).

References

[1] J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals, Molding the Flow of Light,* Princeton Univ. Press, N.J., 1995.

[2] Dimitra I. Kaklamani and Hristos T. Anastassiu, "Aspects of the method of auxiliary sources (mas) in computational electromagnetics", *AP-M,* vol. 44, no. 3, pp. 48–64, June 2002.

[3] Chongjiun Jin, Shouzhen Han, Xiaodong Meng, Bingying Cheng, and Daozhong Zhang, "Demultiplexer using directly resonant tunneling between point defects and waveguides in a photonic crystal", *Jounal of Applied Physics,* vol. 91, no. 7, pp. 4771–4773, April 2002.

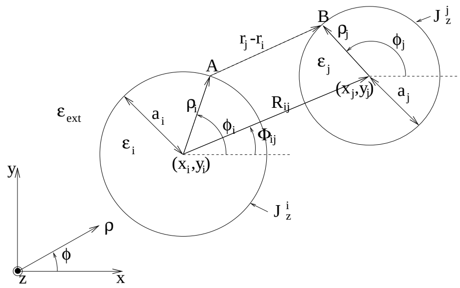

Fig. 1. Basic geometry and coordinate systems

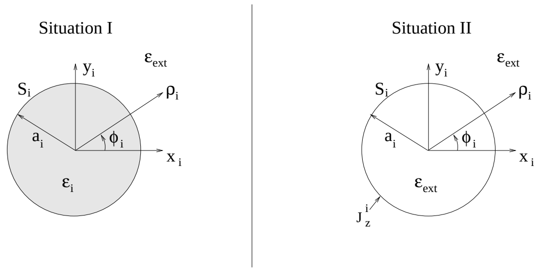

Fig. 2. Two situations for the calculation of the boundary impedance

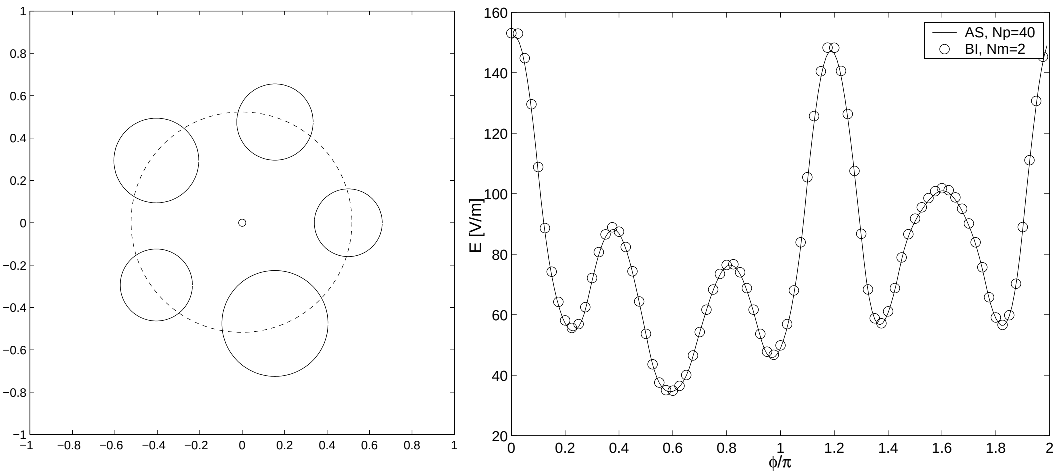

Fig. 3. Comparison of the Boundary Impedance (BI) method with the method of Auxiliary Sources (AS)

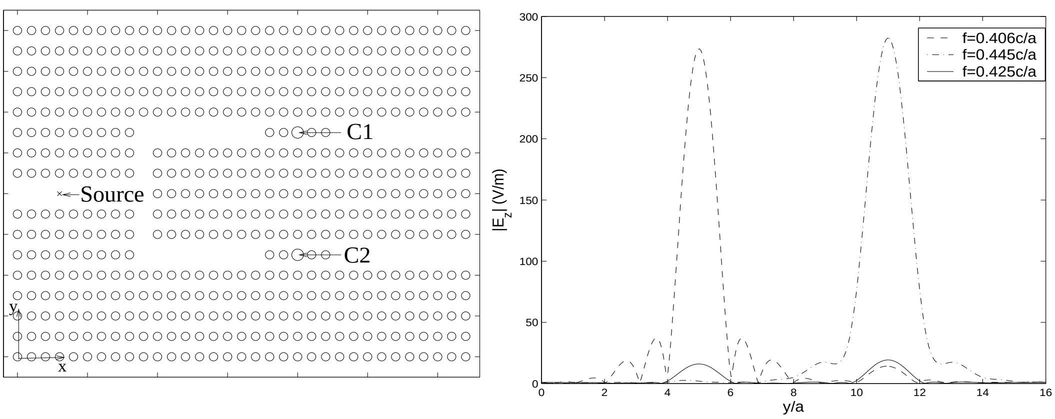

Fig. 4. Example: demultiplexer