Radiation from a Short Vertical Dipole in a Disk-Type PBG Material †

Mário G. M. V. Silveirinha*, Carlos A. Fernandes

Instituto Superior Técnico, Instituto de Telecomunicações
Av. Rovisco Pais 1049-001 Lisboa, Portugal, mario.silveirinha@co.it.pt

1. Introduction

A photonic crystal has in general directional band-gaps where the propagation of electromagnetic waves is inhibited. An antenna in a photonic crystal cannot radiate energy into the directions of space in the band-gap regime. Consequently, the radiation into the other directions can be greatly enhanced and the gain can be very high. Several antenna applications have been proposed in the recent years that explore this concept (see for example [1]).

The radiation of electromagnetic waves in unbounded periodic structures is investigated in [2-3]. In [2] the power radiated by an electric dipole located in a dielectric crystal with air spheres is calculated. The results show the total inhibition of emission in the photonic band gaps, and a strong enhancement near the band edges. It is verified that the radiated power is strongly dependent on the dipole position in the photonic crystal.

Some fundamental aspects of the radiation problem in unbounded photonic crystals were not examined in previous studies. These include the convenient treatment of the radiation off the band gaps, and the calculation of the far field. In [4] we present a detailed analysis of these topics, and in [5] we obtain the radiation field from a line source in the wire medium. In this paper, we calculate the far field from a short electric dipole in a disk-type medium. We discuss the physics of the radiation process in the periodic structure.

The motivation for the study is to enlighten the physics of the radiation mechanism in metallic crystals, and assess the effect of the metallic crystal on the radiation characteristic for long wavelengths. This latter topic is of particular interest in the design of artificial material lenses. Indeed, it has been a common practice in highly shaped beam lenses to embed the feed element into the lens body. It is thus of obvious interest to assess the implications and limitations of proceeding in the same way in artificial material lenses.

2. Geometry and Formulation

The disk-type medium consists of a three-dimensional array of PEC metallic disks embedded in air. We admit that the lattice primitive vectors are $\mathbf{a}_1 = (a_{//}, 0, 0)$, $\mathbf{a}_2 = (0, a_{//}, 0)$ and $\mathbf{a}_3 = (0, 0, a_{\perp})$. The primitive-vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ are parallel to the metallic disks. The geometry of the problem is sketched in Fig. 1.

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The field radiated from a short electric dipole in the metallic crystal is characterized by the electric Green dyadic $\overline{\overline{\mathbf{G}}}_0(\mathbf{r},\mathbf{r}';\mathbf{k})$. The dyadic satisfies:

$$\nabla\times\nabla\times\overline{\overline{\mathbf{G}}}_0 = \beta^2\,\overline{\overline{\mathbf{G}}}_0 + \overline{\overline{\mathbf{I}}}_d\,\delta(\mathbf{r}-\mathbf{r}') \tag{1a}$$

$$\hat{\mathbf{v}}\times\overline{\overline{\mathbf{G}}}_0 = 0, \qquad\qquad \mathbf{r}\in\partial D_{\mathbf{I}} \tag{1b}$$

where $\hat{\mathbf{v}}$ is unit vector normal to the **I**-th metallic surface $\partial D_{\mathbf{I}}$, $\beta=\omega/c$ is the wave number, ω is the angular frequency, and c is the velocity of light in vacuum. The Green dyadic can be constructed by superimposing the fields from phase-shifted arrays of short-dipoles [2, 3]. In [4] we prove that the asymptotic form of the Green dyadic is:

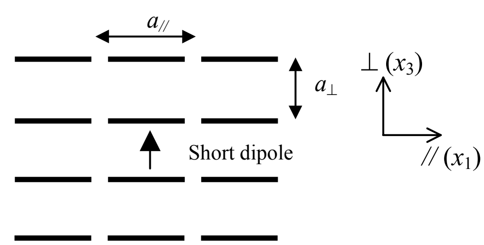


Fig. 1 A short vertical dipole in the disk-type medium.

$$\overline{\overline{\mathbf{G}}}_0 \doteq \sum_n \sum_{\mathbf{k}_{st}\in \mathsf{S}^{(n)}(\beta^2)} \frac{\mathbf{E}_n(\mathbf{r};\mathbf{k}_{st})\mathbf{E}_n^*(\mathbf{r}';\mathbf{k}_{st})}{2\pi|\mathbf{r}-\mathbf{r}'|\left|\nabla\beta_n^2\right|}\frac{\mathrm{V}_{\mathrm{cell}}}{\sqrt{|K_1K_2|}}\, e^{j\frac{\pi}{4}\left(\sum_{r=1}^{2}\mathrm{sgn}(K_r)-2\right)} \tag{2}$$

In the above, $\mathbf{E}_n$, n=1,2,..., are eigenmodes of the Hermitian operator $\nabla\times\nabla\times$. The eigenmode $\mathbf{E}_n(\mathbf{r};\mathbf{k})$ is a Floquet wave with wave vector **k**. For a given **k**, the eigenmodes form a complete orthonormal set over the unit cell Ω. The eigenvalue associated to the mode $\mathbf{E}_n$ is β_n^2. We have that:

$$\int_\Omega \mathbf{E}_n^*(\mathbf{r}).\mathbf{E}_m(\mathbf{r})d^3\mathbf{r} = \delta_{n,m} \qquad\qquad n,m=1,2,... \tag{3a}$$

$$\nabla\times\nabla\times\mathbf{E}_n(\mathbf{r};\mathbf{k}) = \beta_n^2(\mathbf{k})\mathbf{E}_n(\mathbf{r};\mathbf{k}) \tag{3b}$$

$$\hat{\mathbf{v}}\times\mathbf{E}_n = \mathbf{0}, \qquad\qquad \mathbf{r}\in\partial D_{\mathbf{I}} \tag{3c}$$

In (2), the outer sum represents a sum over all bands, whereas the inner sum represents a sum over the stationary points of the n-th band (if any). The stationary phase points are in the wave normal surface $\mathsf{S}^{(n)}(\beta^2) = \{\mathbf{k}\in \mathrm{BZ} : \beta_n^2(\mathbf{k}) - \beta^2 = 0\}$, where BZ denotes the Brillouin zone of the reciprocal lattice. A point $\mathbf{k}_{st}$ in $\mathsf{S}^{(n)}(\beta^2)$ is stationary if and only if the observation direction $\hat{\mathbf{u}}$ is directed and oriented as $\nabla_{\mathbf{k}}\beta_n^2$ at $\mathbf{k}_{st}$. In (2), K_1 and K_2 are the curvatures of surface $\mathsf{S}^{(n)}(\beta^2)$ at the stationary phase point $\mathbf{k}_{st}$ (the normal to $\mathsf{S}^{(n)}(\beta^2)$ is oriented as $\nabla_{\mathbf{k}}\beta_n^2$). $\mathrm{V}_{\mathrm{cell}}$ is the volume of the unit cell.

Each term of the asymptotic formula (2) is the modulation of a slow varying decaying term by an electromagnetic Floquet mode. The Floquet mode reproduces the local sharp fluctuations of the radiation field nearby the metallic inclusions. The effect of the source position on the radiation field is also expressed in terms of a Floquet mode. The radiation pattern associated to the field $\overline{\overline{\mathbf{G}}}_0.\hat{\mathbf{u}}_m$ is [4],

$$U^{(m)} = \sum_n \sum_{\mathbf{k}_{st} \in S^{(n)}(\beta^2)} \frac{1}{16\pi^2 \beta Z_0} \frac{\left|\mathbf{E}_n(\mathbf{r}';\mathbf{k}_{st}).\hat{\mathbf{u}}_m\right|^2 \mathrm{V}_{\mathrm{cell}}}{\left|\nabla \beta_n^2\right|\left|K_1 K_2\right|} \tag{4}$$

where Z_0 is the impedance of free-space.

3. Results and Conclusions

In this section, we present some examples that illustrate the radiation from a short-vertical dipole in the disk-type medium. The disk area fraction (relative to the transversal lattice) is $f_A = 50\%$. We admit that $a_\perp = 0.75 a_{//}$. The short dipole is normal to the metallic disks (i.e. oriented in the $\perp$-direction). The band structure of the disk-type medium is calculated using the generalization of the hybrid method proposed in [6]. For large wavelengths, only the two fundamental bands contribute to the radiation pattern. One band is associated to quasi-E-polarization modes (the average magnetic field is parallel to the disks), whereas the other band is associated to quasi-H-polarization modes (the average electric field is parallel to the disks).

The main contribution to the radiation field due to the vertical dipole is from the quasi-E-polarization modes. The contribution from the H-polarization modes is residual. The contour plot of the dispersion characteristic of the E-polarization modes, $\beta = \beta(\mathbf{k})$, is depicted in Fig. 2. The contours correspond to different wave normal surfaces (the contours in the figure correspond to - from the inner region to the exterior - $\beta a_{//}$=0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5). The rectangular region depicted in the figure is the intersection of the Brillouin zone with the plane $k_2 = 0$. The points shown in the Figure are $\mathrm{Z} = (0,0,\pi/a_\perp)$, $\Gamma = (0,0,0)$ and $\mathrm{X} = (\pi/a_{//},0,0)$. The Floquet modes that contribute to the far field are in the wave normal surface $\beta = \beta(\mathbf{k})$.

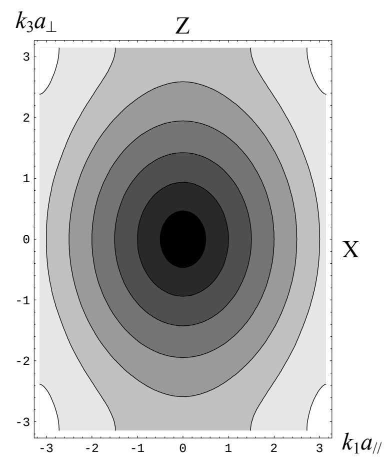
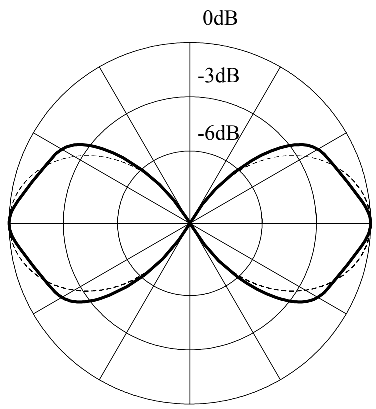
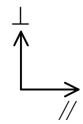


Fig. 2 Contour plot of the dispersion characteristic of the quasi-E-polarization modes in a disk-type medium.

The radiation pattern from an infinitesimal vertical dipole is very sensitive to the source position, since the amplitude of the electric eigenmode in (4) also is. An infinitesimal dipole is a mathematical abstraction. For long wavelengths, it is more realistic to investigate the radiation from a dipole that is short as compared with radiation wavelength, but not necessarily short as compared with the unit cell dimension. The radiation pattern of such antenna is obtained by replacing the term $\mathbf{E}_n(\mathbf{r}';\mathbf{k}_{st}).\hat{\mathbf{u}}_m$ in (4) by the corresponding spatial average in the source coordinates (i.e. the density of current is assumed uniformly distributed in the unit cell). In this paper, we consider that such situation holds.





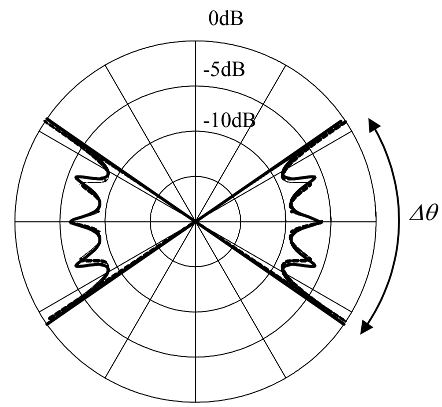


Fig. 3 Radiation pattern for $\beta a_{//}$=0.5 (dashed line) and $\beta a_{//}$=2.5 (full line).

Fig. 4 Radiation pattern for $\beta a_{//}$=3.0 (full line: ZΓX plane; dashed line: ZΓM plane). The width of the radiated beam is $\Delta\theta$.

In Fig. 3 we depict the radiation pattern for the normalized wave numbers $\beta a_{//} = 0.5$ (dashed line) and $\beta a_{//} = 2.5$ (full line). The corresponding wave normal surfaces are perturbed ellipsoids (see Fig. 2). The radiation pattern for $\beta a_{//} = 0.5$ (and more generally for large wavelengths) is coincident with the radiation pattern from a short-dipole in the homogenized medium. The homogenized medium is characterized by the static permittivity $\varepsilon_{r//} = 1.53$ and $\varepsilon_{r\perp} = 1$. For long wavelengths, the radiation pattern is axis-symmetric relatively to the ⊥-direction. The disk inclusions favor the radiation into the radial (//) direction. Indeed, intuitively the disks tend to block the radiation into the ⊥-direction. The radiation pattern for $\beta a_{//} = 2.5$ does not differ significantly from the long-wavelength limit case. For frequencies slightly larger than $\beta a_{//} = 2.5$ a directional band-gap emerges. The wave normal surface for $\beta a_{//} = 3.0$ is depicted in Fig. 2. Now the surface differs significantly from an ellipsoid. There are no stationary wave vectors for directions close to the ⊥-direction. The radiation intensity in the ZΓX-plane (full line) and in the ZΓM-plane (dashed line) are depicted in Fig. 4 (the M point is $\mathrm{M} = \left(\pi/a_{//}, \pi/a_{//}, 0\right)$). The results are practically coincident, and so the radiation pattern is approximately axis-symmetric. The width of the radiated beam is $\Delta\theta = 69^{\mathrm{o}}$ in the ZΓX-plane, and $\Delta\theta = 67^{\mathrm{o}}$ in the ZΓM-plane. The radiation intensity vanishes identically in the cone region that contains the ⊥-direction (i.e. in the directional band gap). The radiation intensity is extremely large near the cut-off directions. In fact, the radiation pattern diverges to infinity at these directions because the Gaussian curvature of the wave normal surface vanishes at the stationary point. The radiated power remains finite.

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