# A REVIEW OF FREQUENCY AND TIME DOMAIN HYBRID RAY-MODE REPRESENTATIONS

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When time-harmonic wave propagation is spatially (transversely) constrained by physical impenetrable boundaries or by “virtual” boundaries (ducts) established through refraction in transversely inhomogeneous media, the resulting source-excited longitudinally guided (ducted) waves have traditionally been described in terms of two alternative phenomenologies: progressing and oscillatory. The progressing formulation views the wavefields as continuous spectra of waves which propagate away from the source to the receiver, through the guiding environment, via multiple transverse reflections and (or) refractions. The oscillatory formulation views the wavefields as discrete (or discrete-continuous) superpositions of frequency-dependent (guided mode) eigenspectra which are matched to the entire transversely confining cross section, which are individually independent of source and receiver locations, but whose *amplitudes* of excitation and reception do depend on these locations.

At high frequencies (HF) – the “overmoded” regime where the transverse dimensions span many wavelengths and many modes can propagate – the oscillatory formulation becomes unwieldy, and is not well-matched to the wave physics, for *large* source-receiver separations since the lower mode eigenspectra there form closely spaced clusters. Here, the progressing wave spectra can be more efficiently adapted to the wave physics by approximate tracking of *local* plane waves whose constructive interference maxima coalesce around source-receiver dependent “ray” trajectories which are determined via HF asymptotics. On the other hand, at *small* source-receiver separations, the progressing formulation becomes unwieldy because it entails many closely spaced multiple reflections, whereas the oscillatory lower eigenmode spectra are widely spaced and, while excited in their totality near the source, decay exponentially (are evanescent) longitudinally away from the source when the number of oscillations in their transverse profile exceeds a frequency-dependent cutoff threshold.

These circumstances have motivated efforts to combine these two *complementary* spectral methodologies, neither of which is convenient for *all* source-receiver locations, in a manner that seeks to exploit the best features of each. The outcome has been a comprehensive rigorously based, selfconsistent *hybrid ray-mode algorithm*, which has clarified the ray-mode interplay through a series of spectral studies and wide-ranging applications to complex waveguiding environments initiated more than two decades ago. These studies are reviewed here because of their potential relevance to HF interaction with complex “new” environments that combine both ray-adaptable and mode-adaptable constituents. Also discussed are corresponding time domain short-pulse-excited hybrid wavefront-resonance schemes. Illustrative examples are included in the presentation.