A Frame Based Beam Summation Algorithm for Wideband Radiation

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Beam-based discrete phase space algorithms are an important tool for tracking high frequency wave fields since they provide a systematic framework for ray-based solutions in complex configurations, e.g., slowly or abruptly varying inhomogeneous media. In these algorithms a given aperture field is expanded into a set of spatially and spectrally shifted expansion functions. When propagating away from the aperture, each expansion function gives rise to a shifted and tilted beam, which propagates along its corresponding ray trajectory. The radiated field is obtained as a sum of all beam contributions at the observation point.

In a recent paper, we presented a Windowed Fourier Transform (WFT) frame-based wideband beam representation that overcomes the difficulties of the critically complete Gabor expansion (e.g., window's non-locality and a frequency dispersion of the beam lattice). Using an overcomplete set of expansion functions adds a degree of freedom, which makes it possible to use the same beam lattice for all frequencies. The overcompleteness also smoothes out the dual (analysis) function and thereby localizes and stabilizes the expansion coefficients. Using a properly chosen set of iso-diffracting Gaussian-beam basis functions also provides the "snuggest", i.e. numerically, the most efficient and compact representation for all frequencies.

The emphasis in the presentation is on analytical study of the expansion and beam parameters that render the beam representation most effective and localized. It is shown that the frequency band which can be efficiently treated with a given beam lattice is of order of one octave in width. For wider excitation bandwidths, the aperture field is sub-band filtered into a hierarchy of one octave wide frequency bands. Each band is then treated separately with its own beam parameters. The beam parameters of different bands are related by a decimation scheme that leads to a self consistent representation wherein the beam axes of the highest frequency band are partially reused for all the other bands.

These algorithms will be applied to a 3D numerical modeling of a wide-band radiation from a focused aperture. We shall show that a phase-space localization greatly reduces that number of beams contributing to the field at any given location, thus leading to a numerically efficient algorithm. It will also be shown that the field calculated using these algorithms is uniform upon transition through caustics and cusps. A time domain counterpart of the algorithm is presented in a companion paper [1].

[1] A. Shlivinski, E. Heyman, A. Boag and C. Letrou, "A pulsed beam algorithm for transient radiation from extended apertures," *This Session* .