"Undergraduate Electromagnetics for the Future: Discrete, Numeric, and Analytic"

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ABSTRACT – For the past six years the first course in engineering electromagnetics at Rose-Hulman has focused on the standard lumped elements of circuits—resistors, capacitors, and inductors. The course coverage is limited to quasi-static, 2-D Cartesian geometries with no leakage flux—conditions that are descriptive of many ideal, lumped circuit elements with which the students are already familiar. These limitations reduce the complexity of the field expressions, yet retain the essential features of multi-dimensional fields. In addition, spatial discretization and numeric methods provide the foundation from which the properties and behavior of electromagnetic vector fields are developed. Most importantly, the duality of the fields in these lumped elements enables the use of Laplace's equation to describe the fields in all three elements. This formulation in discretized form provides the basis for numeric computations that exploit the power of modern PCs to provide a rich graphic environment that enables students to display the solutions in a variety of visual modes. The simplicity of this approach allows students to focus upon the electromagnetic principles and to gain valuable experience in solving practical problems. Typical classroom and homework activities include the use of spreadsheets, matrix solvers, and circuit simulators. In addition, a powerful, PC-based, interactive, real-time, electromagnetic computation engine, VEM2.0, has been developed that empowers students to experiment with the geometry and material properties of 2-D electromagnetic problems. As the course progresses and students gain experience and confidence, the more traditional formulations of continuous differential and integral vector calculus are introduced to complete their preparation for advanced courses. This technologically intensive approach is particularly attractive since all of our students have laptops with wireless networking. Course details and software demonstrations are included in the presentation.

INTRODUCTION

Nearly 10 years ago we conducted a critical review of our curriculum with an eye toward modernization of the approach, content, and delivery of our courses. The required, core courses (and electromagnetics in particular) were scrutinized as to their importance and relevance in a rapidly changing technical world. Fortunately, these deliberations lead to a strong affirmation of the importance of electromagnetics in our curriculum. However, we felt that the content and method of delivery required changes to meet the today's needs. Two key issues in our discussions focused upon the **abstract nature and challenges of vector calculus** and the **technological tools** made available by powerful, low-cost PCs. We decided to start with a clean slate and to re-design our introductory electromagnetics course. We have been teaching the resulting course for six years and find that it provides students a strong connection between the fields and the mathematics. Moreover, this approach utilizes the powerful computing tools available to today's students.

THE APPROACH

A significant departure from traditional electromagnetics was chosen in order to enhance our students' understanding of electromagnetics. Implicit in this approach is a change in the presentation of electromagnetics to students; learning is focused on the concepts and tools used by today's electromagnetic professionals. Two fundamental principles are synergistically combined to form the foundation of this approach--**spatial discretization** and **numeric formulation**. Without numeric formulations that can exploit computational power and graphics, the concepts of spatial discretization offer an interesting, but limited, alternative to continuous electromagnetics. Without spatial discretization, numeric methods are limited to formula evaluation and plotting of solutions.

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Spatial Discretization

Our approach departs from the traditional vector calculus of continuous mathematics by considering space to be divided into discrete incremental cells each of that can be modeled as a lumped circuit element. This construct allows the laws of electromagnetics to be cast in terms of lumped element circuit theory—resistance, capacitance, inductance, Ohm's law, Kirchoff's voltage and current laws. Basic field concepts are developed within the familiar framework of the terminal behavior of circuit elements. The axioms of this approach are based in circuit theory rather than in physical observations and vector calculus. The vector description of fields follows naturally from the behavior of incremental circuit elements. Spatial variations of fields are manifest in the geometric arrangement of the incremental cells. Macroscopic element values follow directly from the rules for series and parallel elements of circuits and lead intuitively to the more traditional line and surface integral representation of continuous-variable, vector calculus.

Numeric Formulation

The elegance and rigor of continuous-variable vector calculus of electromagnetics have great appeal to many professionals (and most textbook writers) and with good reason—they are a necessity for advanced concepts of electromagnetics. On the other hand, spatial discretization, in addition to its basis in circuit theory, allows the use of computer-oriented (and less intimidating) discrete mathematics. Formulation of electromagnetic laws in discrete form leads beginning students directly to numeric-based solutions. Real-time, scalar calculations displayed with color graphics provide a powerful learning environment. Using this process, students can understand the basic principles of electromagnetics before they master differential and integral calculus. This numeric formulation enables students to grasp more readily the physical foundations of electromagnetics. Moreover, it heightens the students'awareness of the value of vector calculus. An intuitive sense of electromagnetic field behavior can be acquired by computer experimentation. An added benefit is that numeric formulations enable students to obtain computer solutions to practical problems. Students can solve "real world" problems, not just those which satisfy the artificial boundary geometry of infinite planes, infinite circular cylinders, or spheres.

THE DETAILS

Spatial discretization is a powerful method by which the internal electromagnetic details of circuit elements are readily visualized and related to the terminal behavior of lumped circuit elements. Several assumptions are used in this approach in order that the fundamental features of the fields are not hidden by unnecessary, more advanced details.

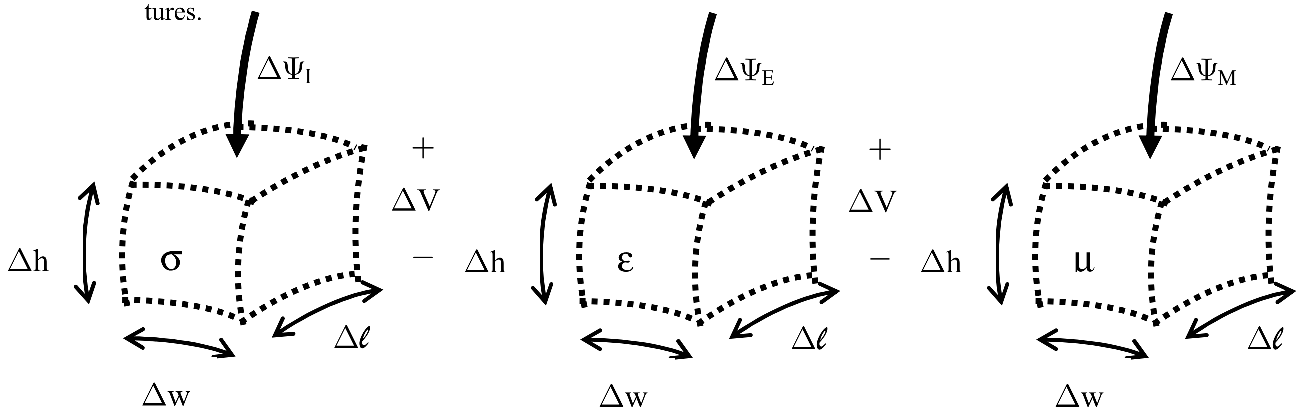
In this approach, a student's first encounter with electromagnetic fields is limited to **quasi-static cases** so that the structures are small compared to wavelength corresponding to the conditions of lumped elements. But more importantly, the complexities high-frequency, radiating fields are delayed until students have some experience with electromagnetics. **The underlying principles are found in circuit theory and elementary concepts of freshman physics**. For example, the variation of voltage along a straight wire leads to the slope of voltage with distance $\Delta V/\Delta x$ to give local variation. In the limit this becomes dV/dx. This is generalized to three dimensions and the definition of the **gradient**. Or, the application of KCL in circuits to an incremental cube leads to the definition of **divergence**. Physical intuition and generalization leads to conservation of charge.

Cartesian coordinates are chosen for most of the course in order to minimize the masking of electromagnetic principles by the complexities of new coordinate systems. Most structures are defined by or can be adequately approximated in Cartesian coordinates so that only a limited number of applications in cylindrical and spherical coordinates are included.

Further simplification is gained by considering mainly **two-dimensional structures**. One-dimensional structures provide a simplistic view of fields limited to only a single spatial variable and vector direction. Two-dimensional structures are adequate to show the variety of basic properties and spatial variations of potentials and vectors; the added complexity of three-dimensional structures is unnecessary. Equally important, two-dimensional electromagnetic fields quite conveniently take advantage of computer graphics displays.

Structures and solution techniques are simplified by assuming that ideal flux-guiding material is present in all devices so that all of the flux is confined to the material, i.e., there is **no fringing or leakage flux**. The exclusion of fringing flux exactly models the fields of resistors and is a reasonable approximation for most capacitors. However, inductors are limited to closed, magnetic cores with at most a thin air gap. Air core inductors are not covered since the computational complexities associated with leakage flux do not reveal any additional basic principles, only more complex math.

Under these conditions, the behavior of all lumped element devices is governed by $\boldsymbol{\nabla^2 V = 0}$, **Laplace's equation**, where V represents a scalar electric or magnetic potential. The perpendicular nature of equipotential surfaces and flux lines is exploited to describe local behavior in terms of **discretized, incremental elements**. In this representation, all three incremental element values, ΔX, are calculated by the same general form, $\Delta X = \Delta\Psi/\Delta V$, that can be specialized to give numeric values of incremental elements as $\Delta G = \sigma\Delta a/\Delta h$. $\Delta C = \varepsilon\Delta a/\Delta h$, and $\Delta L = \mu\Delta a/\Delta h$ for conductors, capacitors, and magnetic reluctors, respectively. Figure 1 illustrates the relationship of the flux and potential difference of the incremental elements. The incremental elements are combined by the familiar series and parallel procedures of circuit theory to element values for finite structures.



RESISTORS

$$\Delta G = \frac{\Delta\Psi_I}{\Delta V} = \sigma\frac{\Delta w \Delta\ell}{\Delta h} = \sigma\frac{\Delta a}{\Delta h}$$

CAPACITORS

$$\Delta C = \frac{\Delta\Psi_E}{\Delta V} = \varepsilon\frac{\Delta a}{\Delta h}$$

INDUCTORS

$$\Delta L = \frac{\Delta\Psi_M}{\Delta V_M} = \mu\frac{\Delta a}{\Delta h}$$

Figure 1 - Incremental Elements

This approach is **technologically intensive**; analytic tools with a modicum of graphics have been replaced by numeric, highly visual tools. With the ready availability of high-performance computers, today's students are able to numerically and analytically solve a wide variety of electromagnetic problems. The limited power of the calculator, paper, and pencils is replaced by the powerful tools commonly used by electromagnetics professionals. Several examples follow.

EXAMPLES

Throughout the course, attention is focused upon calculating the scalar electric or magnetic potentials and the current, electric, or magnetic flux associated with them. The distribution of potential within arbitrary, 2-D structures is easily calculated by a variety of numeric methods. For homogeneous materials, the discretized form of Laplace's equation provides a simple calculation method.

For inhomogeneous materials computations are based upon the integral form of the flux continuity equation.

A typical example is that of a 2-D, ell-shaped, corner resistor in air. The solution of internal potentials and associated current flow is especially well-suited to spreadsheet applications such as Excel. The well-known relaxation method is implemented via iterative solutions of the finite difference equations within each cell of the resistor. The 2-D footprint of the resistor provides a visual description of the calculation region within the spreadsheet. Of course, the form of each cell equation depends upon the location of the cell—internal, edge, inside corner, or outside corner—and on the adjacent material. The nodal potentials provide the basis for computing the electric field intensity which in turn leads to the current flux density and finally to the total current flux within the resistor. The resistance of the element is simply the ratio of the voltage to current.

Alternatively, the nodal potentials and currents within a resistor can be calculated by a matrix solver such as Matlab or circuit simulator such as PSpice. The duality of the KCL for a circuit node and the discretized form of Laplace's equation is the basis for calculation of the nodal potentials and currents within a resistor, also.

As the students' familiarity with electromagnetic fields increases, the discrete mathematics of the previous examples is supplemented with the continuous mathematics of vector calculus. A powerful symbolic tool with which the students are familiar, MAPLE, is used to reduce the tedium of analytic calculus. Moreover, its powerful display features provide a rich variety of graphics.

Visual ElectroMagnetics (VEM2.0) [1] is a two dimensional, electromagnetic simulator designed and developed as a visualization aid for students in undergraduate electromagnetics. VEM2.0 utilizes finite difference techniques in conductive, electrostatic and magnetostatic environments. The VEM2.0 code, written in MATLAB 5, provides an intuitive, user-friendly graphical interface which is inexpensive and platform independent. VEM consists of a **structure window** in which the user draws the electromagnetic materials and sources in Cartesian or angularly-symmetric cylindrical coordinate systems via common drawing tools and pop-up menus. The **solver button** generates the system matrix, solves it, and activates the **solutions window** in which the results are displayed in a variety of 2-D and 3-D, user-selected viewing modes. VEM2.0 provides virtually real-time results with solution times of less than 10 seconds. Though the solution region is finite in extent, its outer boundaries appear open as if the solution region were of infinite extent. A compact simulation of the open boundaries is implemented via the Transparent Grid Termination (TGT) techniques [2].

CONCLUSION

Our observations indicate that students find this approach more "connected" with the physical world than the traditional approach. They are able to perform calculations and obtain solutions for complicated shapes that model actual elements. Moreover, their performance in the following electromagnetics course indicates that they have a solid understanding of the fundamentals. We are encouraged by these results and are extending this technique to electromagnetic waves.

A demonstration of the VEM2.0 software will be included in the presentation.

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