Electromagnetic Scattering from a Multilayered Cylindrical Waveguide

F. Seydou[1,2], *R. Duraiswami*[2] *& T. Seppänen*[1]

1. Department of Electrical and Information Engineering
University of Oulu, P.O. Box 3000, 90401 Finland
2. Institute for Advanced Computer Studies
University of Maryland, College Park, MD

Abstract

This paper is devoted to electromagnetic scattering from an N multilayered circular cylinder. We consider waveguides in the z direction, that is we look for the solution of Maxwell equations along the z direction. We assume a dielectric core and derive a mode matching approach for solving the problem. A numerical result is presented that illustrates the algorithm .

Introduction

In this paper we discuss some analytical and computational results for the electromagnetic scattering problem from N layered scatterer. This problem is of significant importance in many areas of in microwave and optical applications (see [1] and the references therein). The scatterer is assumed to be a nested body consisting of a finite number of homogeneous layers (annular regions) with a dielectric core. We consider waveguides in the z direction, that is we look for the solution of Maxwell equations along the z direction. This problem is also called the two-and-one-half dimensional problem. An important amount of work was reported for the TM and TE case (see [2]) whereas much less is shown for the more complicated hybrid case where both Debye potentials E_z and H_z are needed to construct solutions in cylindrical waveguides. Here we try and remedy this gap by deriving and solving linear algebraic equations similar to the TE and TM case discussed in [2].

1 Statement of the problem

We are looking for the solution $(\mathbf{E}, \mathbf{H})$ of Maxwell system of equations

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H},$$

$$\nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E},$$

such that

$$\mathbf{F}(x, y, z, t) = \mathbf{F}(x, y)e^{i\omega t - i\beta z},$$

where $\beta \neq 0$ is the wave number along the guide direction and $\mathbf{F}$ stands for $\mathbf{E}$ and $\mathbf{H}$. The latter notation is assumed throughout this paper.

If we write each vector and each operator in the above equations as the sum of a transverse part, designated by the subscript T, and a longitudinal component, we obtain the modified Maxwell system of equations which can be solved to obtain:

$$\mathbf{E}_T = \alpha\left(\nabla_T E_z - \eta\vec{e}_3 \times \nabla_T H_z\right) \quad \text{and} \quad \mathbf{H}_T = \alpha\left(\nabla_T H_z + \tilde{\eta}\vec{e}_3 \times \nabla_T E_z\right). \qquad (1.1)$$

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where $\alpha = -\frac{\imath\beta}{\kappa^2}$, $\eta = \frac{\omega\mu}{\beta}$, and $\kappa^2 = \omega^2\epsilon\mu - \beta^2$, $\tilde{\eta} = \frac{\omega\epsilon}{\beta}$, and

$$\nabla_T^2 F_z + \kappa^2 F_z = 0. \tag{1.2}$$

So, the problem of solving the Maxwell system of equations in the waveguide structure reduces to solving (1.2) and obtain $(E_z,\, H_z)$, then recover $\mathbf{H}_T$ and $\mathbf{E}_T$ from (1.1).

We deduce, from (1.1) that

$$E_\phi = \alpha\left(\frac{1}{\rho}\partial_\phi E_z + \eta\partial_\rho H_z\right), \quad \text{and} \quad H_\phi = \alpha\left(\frac{1}{\rho}\partial_\phi H_z - \tilde{\eta}\partial_\rho E_z\right). \tag{1.3}$$

Using separation of variables it is known that the solution of the Helmholtz equation my be obtained. In particular we have

$$[E_z, H_z](\rho,\phi) = \sum_{n=-\infty}^{\infty}\left(H_n^{(1)}(\kappa\rho)\mathbf{B}_n + J_n(\kappa\rho)\mathbf{A}_n\right)e^{in\phi - i\beta z},$$

where J_n is the Bessel function and $H_n^{(1)}$ is the Hankel function of first kind. The coefficients $\mathbf{A}_n = [a_n, \tilde{a}_n]^t$ and $\mathbf{B}_n = [b_n, \tilde{b}_n]^t$ are to be found using boundary conditions. Here the superscript t stands for transpose.

Using (1.3) we have

$$[\mathbf{E}_\phi, \mathbf{H}_\phi]^t(\rho,\phi) = \sum_{n=-\infty}^{\infty}\left[\mathbf{Q}_{1,n}(\kappa,\rho)\mathbf{A}_n + \mathbf{Q}_{2,n}(\kappa,\rho)\mathbf{B}_n\right]e^{in\phi - i\beta z}, \tag{1.4}$$

where

$$\mathbf{Q}_{1,n}(\kappa,\rho) = \alpha\begin{bmatrix} \frac{in}{\rho}J_n(\kappa\rho) & \eta\kappa J_n'(\kappa\rho) \\ -\tilde{\eta}\kappa J_n'(\kappa\rho) & \frac{in}{\rho}J_n(\kappa\rho)\end{bmatrix} \quad \text{and} \quad \mathbf{Q}_{2,n}(\kappa,\rho) = \begin{bmatrix} \frac{in}{\rho}H_n^{(1)}(\kappa\rho) & \eta\kappa H_n^{(1)'}(\kappa\rho) \\ -\tilde{\eta}\kappa H_n^{(1)'} & \frac{in}{\rho}H_n^{(1)}(\kappa\rho)\end{bmatrix}.$$

Now, Let $\mathbf{D}_l$, $l = 0, 1, \cdots N-1$ be N circular cylinders such that $\overline{\mathbf{D}}_{l-1} \subset \mathbf{D}_l$, $l = 1, 2, \cdots, N-1$. Let Γ_l be the boundaries of $\mathbf{D}_{l-1}$, $l = 1, \cdots, M$. Now let $\Omega_1 = \mathbf{D}_0$, $\Omega_l = \mathbf{D}_l \backslash \overline{\mathbf{D}}_{l-1}$, $l = 1, \cdots, N-1$, and $\Omega_M = \mathbf{R}^2 \backslash \overline{\mathbf{D}}_{M-1}$. We assume that Ω_M is simply connected. Each of the regions Ω_l is a dielectric material of complex permittivity and permeability ϵ_l and μ_l $(l = 0, \cdots, M)$, respectively and let the field $\mathbf{F}$ in Ω_l be denoted by $\mathbf{F}_l := \mathbf{F}_{T,l} + \vec{e}_3 F_{z,l}$. The geometry is assumed to be illuminated by an incident field which is a plane wave $F_z^i = e^{i\kappa\mathbf{r}\cdot\mathbf{d} - i\beta z}$ with direction $\mathbf{d} = (\cos\phi_0, \sin\phi_0)$, and $\mathbf{r} = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$. From the analysis above we see that solving the Maxwell equation in the multilayered scatterer using a waveguide structure yields to solving the Helmholtz equation in each domain Ω_l for F_z, that is,

$$(\nabla^2 + \kappa_l^2)F_{z,l} = 0 \quad \text{in} \quad \Omega_l, \quad l = 0, \cdots, N,$$

The other components $\mathbf{F}_{T,l}$ are found using (1.1). In Ω_M we have a sum of the incident field and a scattered field, that is $F_{z,N} = F_z^s + F_z^i$. The scattered field F_z^s should satisfy the Sommerfeld radiation condition.

The above problem should be solved subject to boundary conditions on the interfaces. For Electromagnetic materials, this requires the continuity of the tangential components of the electromagnetic fields across the interfaces.

2 Boundary conditions and solution of the problem

Let r_{l+1} and $\mathbf{O}_{l+1}$ be the radii and the origins of the cylinders , $l = 0, 1, 2, \cdots, N-1$; then we have the following expansions: For the outermost region we have,

$$[E_z, H_z]^t(\rho_M, \phi_M) = \sum_{n=-\infty}^{\infty} \left(H_n^{(1)}(\kappa_M \rho_M)\mathbf{B}_n^M + J_n(\kappa_M \rho_M)\mathbf{p} \right) e^{in(\phi_M - \phi_0) - i\beta z}$$

and for other regions we have

$$[E_z, H_z]^t(\rho_1, \phi_1) = \sum_{n=-\infty}^{\infty} \left(H_n^{(1)}(\kappa_l \rho_1)\mathbf{B}_n^l + J_n(\kappa_l \rho_1)\mathbf{A}_n^l \right) e^{in\phi_1 - i\beta z}, \quad l = 0, 1, 2, \cdots N-1,$$

where $\mathbf{B}_n^0 = 0$

To enforce the boundary conditions we need that the fields expressed in terms of $X_1 O_1 Y_1$ be translated to $X_l O_l Y_l$ coordinates, for $l = 2, 3, \cdots, M-1$. The addition formula yields

$$[E_z, H_z]^t \mathbf{U}_l(\rho_l, \phi_l) = \sum_{n=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} j_{q-n}(\kappa_l d_{l_1}) \left[H_q^{(1)}(\kappa_l \rho_l)\mathbf{B}_n^l + j_q(\kappa_l \rho_l)\mathbf{A}_n^l \right] e^{iq\phi_l - i(q-n)\phi_{l_1} - i\beta z},$$

where d_{l_1} is the distance between $\mathbf{O}_1$ and $\mathbf{O}_l$ and ϕ_{l_1} is the angle between $\mathbf{O}_1\mathbf{O}_l$ and the x axis.

Now the boundary conditions require that (E_z, H_z) and $(\mathbf{E}_\phi, \mathbf{H}_\phi)$ be continuous on the interfaces. This means that $[E_{z,l-1}, H_{z,l-1}]^t = [E_{z,l}, H_{z,l}]^t$ and $[\mathbf{E}_{\phi,l-1}, \mathbf{H}_{\phi,l-1}]^t = [\mathbf{E}_{\phi,l}, \mathbf{H}_{\phi,l}]^t$ for $l = 1, 2, \cdots N$. From (1.4) and the above expansions we can use the boundary conditions to derive an infinite system of linear equations. The sums have to be truncated, at some number, N_0, to obtain a finite system in the unknowns $\mathbf{A}_n^l$ and $\mathbf{B}_n^l$. This system is solved via a conjugate gradient approach.

We validated our numerical results by computing the fields on the boundaries, thus verifying the boundary conditions. We show here the case for the outermost boundary. In the figures below we compute the fields for three cylinders for different values of μ and ϵ. In figure 1 we plot $|E_{z,3}| + |H_{z,3}|$ in solid line and $|E_{z,4}| + |H_{z,4}|$ against the incident angle ϕ_0 on the outermost boundary. Clearly they are identical and thus verify the boundary condition. We also plot in Figure 2 the difference of the two electromagnetic fields against ϕ_0 and β. It vanishes as expected.

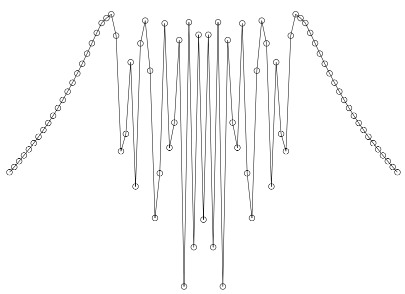

Figure 1: $|E_{z,3}| + |H_{z,3}|$ in solid line and $|E_{z,4}| + |H_{z,4}|$ against the incident angle ϕ_0 on the outermost boundary of 3 layered cylinder.

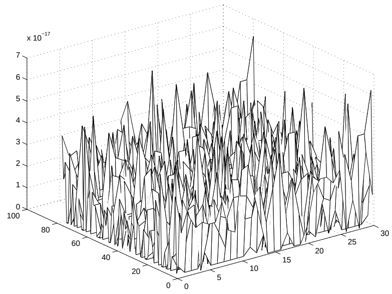

Figure 2: the difference of the two electromagnetic fields against ϕ_0 and β.

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