Integral Equation-Based Coupled Electromagnetic-Circuit Simulation in the Frequency Domain

Yong Wang, Dipanjan Gope*, Vikram Jandhyala, and C.J. Richard Shi

Dept of Electrical Engineering, University of Washington, Seattle WA 98195
Email: jandhyala@ee.washington.edu, Ph: 206-543-2186, Fax: 206-543-3842

In this paper, an approach to couple electromagnetic surface integral equations and circuit simulation is presented. Terminals are defined that connect lumped circuit models to objects modeled with distributed electromagnetic simulation. A modified form of the charge-current continuity equation is proposed for connectivity at terminals. The resulting scheme enables simultaneous solution of electromagnetic integral equations for arbitrarily-shaped objects and SPICE-like modeling for lumped circuits, and permits design iterations and visualization of the interaction between the two domains.

1. INTRODUCTION

There are several existing methodologies aimed at incorporating EM effects in circuit simulations. One such approach, the partial element equivalent circuit (PEEC) method [1,2] has been developed as a successful means to discretize objects and to directly represent the coupling between the discretized elements using SPICE compatible RLC elements and dependent sources. Due to the dense nature of the interactions and the fact that SPICE is tuned for solving sparse matrices, the direct PEEC method is size limited. To obviate this limit, ongoing work has focused on reduced order models and fast solvers.

Another approach is to directly use a regular Method of Moments (MoM) solver to derive the port parameters. Thereafter, equivalent circuits are generated in conjunction with model reduction methods in order to obtain characteristics at the ports that approximate the frequency-dependent EM simulation results. This approach can become unattractive for a variety of reasons, including ill-defined ports at chip and package level, complexity of modeling for multi-port parameters, and loss of information about the electromagnetic problem once the port model is obtained.

In this paper a new approach to formulating and simulating the coupled EM-circuit problem is presented. The distributed EM effects and the lumped circuit models are formulated in conjunction in one system matrix amenable to fast direct and iterative solutions. Although standard port and terminal models for EM structures can also be generated using the approach discussed herein, it is shown that solving the EM-circuit system simultaneously provides more detailed field information and also obviates the equivalent circuit generation step, thus automating the design flow. In addition, the coupled formulation not only handles circuit excitations, but also the effects of incident electric field radiations on the EM-circuit system. The methodology is inherently hierarchical, with seamless transitions possible between circuit and EM representations depending on the level of detail required.

2. FORMULATION

Consider a conducting object with surface S to be modeled with distributed electromagnetic simulation, connected to arbitrary circuits, excited through voltage and

current sources within the circuit, and optionally illuminated by one or more electromagnetic wave excitations. The surface S is divided into two sub-surfaces, denoted by S_{CK} and S_{EM}, where S_{CK} denotes *terminals*, the regions where lumped circuits are connected to S. On the entire surface S the boundary condition for the electric field is:

$$\left[\mathbf{E}^s(\mathbf{J}) + \mathbf{E}^i \right]_{\tan} = Z_s \mathbf{J} \tag{2.1}$$

where $\mathbf{E}^s$ is the scattered electric field produced by the induced equivalent surface current $\mathbf{J}$, $\mathbf{E}^i$ is the incident electric field, subscript tan denotes the tangential components on the surface S, $Z_s = \sqrt{j\omega\mu/2\sigma}$ represents surface impedance. On S_{EM} the standard continuity equation relating the surface current $\mathbf{J}$ and surface charge ρ holds:

$$\nabla_s \cdot \mathbf{J}(\mathbf{r}) + j\omega\rho(\mathbf{r}) = 0 \quad \forall \mathbf{r} \in S_{EM} \tag{2.2}$$

where ∇_s represents surface divergence, with standard notation used for material parameters and frequency above. On S_{CK}, the following condition is proposed. The current flowing out of the circuit node associated with a terminal flows into the patches on S_{CK}. This coupling current introduces an additional source term that alters the surface current and surface charge on S through a modified continuity equation valid for S_{CK}. The modified continuity equation has the following form:

$$\nabla_s \cdot \mathbf{J}(\mathbf{r}) + j\omega\rho(\mathbf{r}) = J_c^m(\mathbf{r}) \quad \forall \mathbf{r} \in S_{CK} \tag{2.3}$$

where J_c^m represents the scalar current density into the terminal m. Therefore the scattered electric field can be expressed in a modified form as:

$$\begin{aligned}
\mathbf{E}^s(\mathbf{J}) = &-j\omega\frac{\mu}{4\pi} \int_S \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}ds' - \nabla\frac{1}{j\omega 4\pi\varepsilon} \int_S \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}[-\nabla_s.\mathbf{J}(\mathbf{r}')]}{|\mathbf{r}-\mathbf{r}'|}ds' \\
&-\nabla\frac{1}{4\pi\varepsilon} \sum_{m=1}^{M} \int_{S_{CK}^m} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}J_c^m(\mathbf{r}')}{j\omega|\mathbf{r}-\mathbf{r}'|}ds'
\end{aligned} \tag{2.4}$$

While other approaches such as delta gap methods or wire basis functions can also be used to describe the coupling, they are either not general enough for connection with arbitrary circuits or need artificial parameters such as basis lengths, directions and radii.

Based on the assumption that the scalar potential produced on electrically small terminals S_{CK}^m is equal to the voltage of the circuit node associated, an additional set of equations can be setup:

$$V_n = \frac{1}{4\pi\varepsilon} \left(\int_S \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}ds' + \sum_{m=1}^{M} \int_{S_{CK}^m} \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}J_c^m(\mathbf{r}')}{j\omega|\mathbf{r}-\mathbf{r}'|}ds' \right), \quad \forall \mathbf{r} \in S_{CK}^n, \quad n=1,\dots,M \tag{2.5}$$

In addition, the regular Kirchoff's Current Law (KCL) is enforced at each terminal circuit node n, which involves all circuit currents entering or leaving the node in addition to the coupling currents.

With the above coupling schemes, and using Rao-Wilton-Glisson basis functions, the coupled problem can be formulated as:

$$\begin{pmatrix} \overline{\mathbf{Z}}_{11} & \overline{\mathbf{Z}}_{12} & \overline{\mathbf{0}} \\ \overline{\mathbf{Z}}_{21} & \overline{\mathbf{Z}}_{22} & \overline{\mathbf{C}} \\ \overline{\mathbf{0}} & \overline{\mathbf{C}}^{\mathrm{T}} & \overline{\mathbf{MNA}} \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{I}_c \\ \mathbf{ckt} \end{pmatrix} = \begin{pmatrix} \mathbf{ex}_{\mathbf{EM}} \\ \mathbf{0} \\ \mathbf{ex}_{\mathbf{ckt}} \end{pmatrix} \tag{2.6}$$

$\overline{\mathbf{Z}}_{11}$ and $\overline{\mathbf{Z}}_{12}$ represent the tested electric field produced by the scalar and vector potentials due to EM currents, and by the scalar potential due to coupling currents, respectively. Similarly $\overline{\mathbf{Z}}_{21}$ and $\overline{\mathbf{Z}}_{22}$ represent the tested scalar potential at the terminal patches produced by the charge associated with the RWG currents and the coupling currents, respectively. $\overline{\mathbf{C}}$ is the EM-circuit connectivity matrix with one non-zero entry per row. $\overline{\mathbf{MNA}}$ represents the Modified Nodal Analysis for the lumped circuit elements. $\mathbf{I}$ and $\mathbf{I_c}$ are the strengths of RWG currents and the coupling currents respectively while **ckt** represents the circuit voltage and current unknowns. The excitations on the RHS include $\mathbf{ex}_{EM}$, the tested incident electric fields and $\mathbf{ex}_{ckt}$, the current and voltage sources.

3. NUMERICAL RESULTS

One typical application is circuit/layout co-simulation for RF electronics systems where on-chip inductors are often employed. Figure 2a shows the topology of a 5.6GHz differential mode Low Noise Amplifier (LNA) where several on-chips inductors are included either for frequency selection purpose (L1 L2) or for impedance matching purpose (L3, L4, L5, L6).
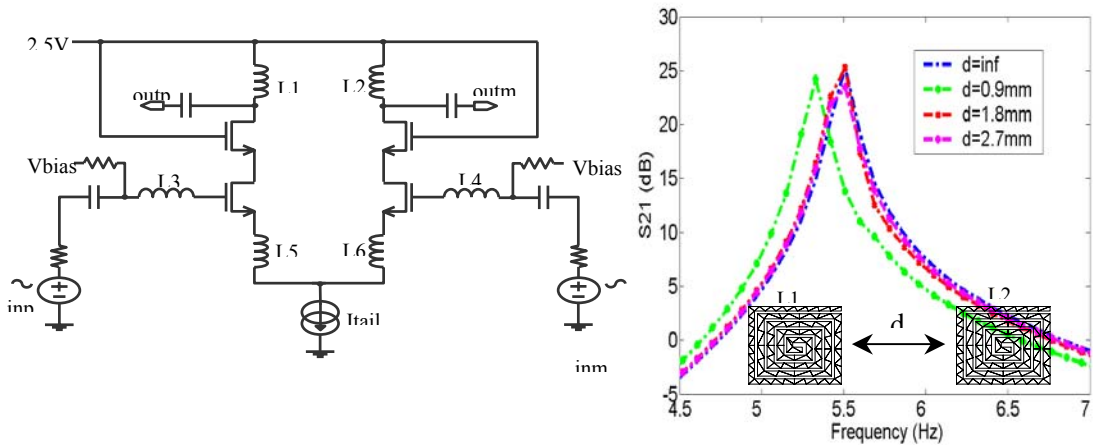

Fig 1a: A 5.6GHz Low Noise Amplifier   Fig 1b: S21 curve versus distance between inductors

Shown in Figure 1b are a series of S21 curves plotted by varying the distances between two inductors L1 and L2, and the resultant shift in center frequency due to changing mutual coupling effects; d=inf corresponds to ignoring mutual coupling. In the above design example, the coupled circuit-EM solver avoids the port model generation/curve fitting steps, which are necessary for traditional design methods.

The second example is to study the power/ground plane voltage bounce distribution due to a high-speed noise source. Consider a typical mixed analog/digital PCB board as shown in Figure 2.
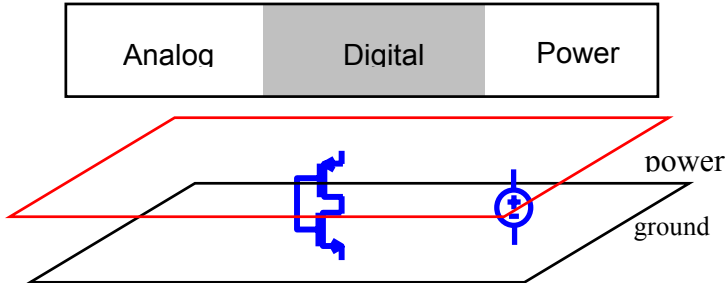

Fig 2: Mixed Signal PCB

With traditional port model based EM-circuit simulation methods, it is difficult to know the bounce voltage distribution all over the plane since voltage/field spatial distribution information is lost in a port model. If all the spatial points on the plane are treated as ports, then the scale of the problem will be extremely large. On the other hand, since the coupled circuit-EM solver uses equivalent surface current as system unknowns, the voltage/field distribution can then be easily derived by a single post processing once the coupled system is solved. Figure 3a shows the ground bounce voltage distribution for a PCB board of dimensions 12cm X 8cm, at 3GHz, with a 1mA noise source.
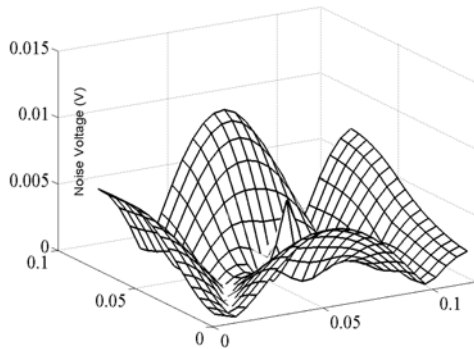

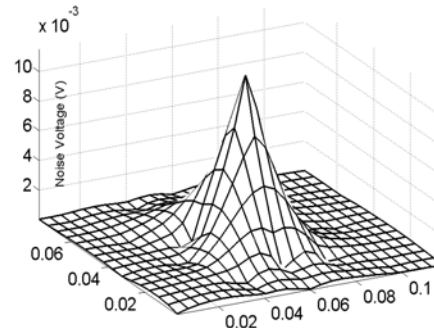Fig 3a: Bounce voltage distribution at 3GHz

Fig 3b: Bounce voltage distribution at 3GHz after 20 decoupling capacitors

By continuously pinning down the peak bounce voltage using 10nF decoupling capacitors, the distribution of noise voltage was controlled as in Figure 3b after adding approximately twenty capacitors. The coupled nature of the proposed approach enabled field-based placement of decoupling capacitors, which would be difficult with port-based methods.

4. CONCLUSIONS

This paper presents a coupled EM-circuit approach that enables seamless transition between the two domains. Although the proposed method can be used to generate frequency-dependent port models, it also permits fully-coupled simulation, thereby automating design flow, and retaining EM information which can provide design insight.

REFERENCES
[1] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagation,* vol. AP-30, pp. 409-418, May 1982.
[2] A. E. Ruehli, "Equivalent circuit models for three-dimensional multiconductor systems," *IEEE Trans. on Microwave. Theory and Techniques.,* vol. MTT22 (3), pp. 216-221, March 1974.
[3] K. Kundert, H. Chang, D. Jefferies, G. Lamant, E. Malavasi, and F. Sendig, "Design of mixed-signal systems-on-a-chip," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems,* vol. 19 no.12, pp. 1561–1571, Dec. 2000.
[4] W. Pinello, A. C. Cangellaris, and A. Ruehli, "Hybrid electromagnetic modeling of noise interactions in packaged electronics based on the partial-element equivalent-circuit formulation," *IEEE Trans. on Microwave Theory and Techniques,* vol. 45, no.10 part 2, pp. 1889–1896, Oct. 1997.