Spectral Methods in General Curvilinear Simplex Grids

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The ever-increasing need for the modeling of large-scale broadband electromagnetic problems has attracted intensive research in high-order and spectral time-domain methods. Examples of such methods are the pseudospectral time-domain methods, and high-order finite-element time-domain method, high-order discontinuous Galerkin method. Compared to the conventional finite-difference time-domain method, high-order and spectral methods are attractive because of their high efficiency and accuracy. However, so far most spectral methods have only used elements with straight faces, thus limiting the accuracy for curved geometries. In this work, we develop a spectral method based on a high-order simplex grid so that the element faces can be curved. With this discretization, the accuracy of the geometrical representation is compatible with that of the field interpolation scheme. The effects of discretization approximation on the accuracy of spectral penalty methods are studied for general high-order curved elements. For numerical implementation, the spectral method for 3D Maxwell's equations is developed with a quadratic simplex grid. Numerical results show the accurate discretization of the object geometry plays an important role in the simulation with spectral methods. The solution for the second-order discrete approximation of the quadratic simplex grid is much more accurate than that for the first-order discretization approximation of the tetrahedron grid. Furthermore, this quadratic simplex spectral method is more efficient that traditional linear simplex grid since it allows a much coarser grid for curved objects.