Iterative Preconditioning of Three-Dimensional FE-BI Equation Systems

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The finite element (FE) - boundary integral (BI) technique provides for an exact formulation of three-dimensional (3D) scattering and radiation problems. Early formulations appeared in the late 80es of the last century, however, it was quickly recognized that the equation systems resulting from standard discretizations are rather difficult so solve. Consequently, not many formulations were pursued over the years and later even doubts arose whether standard formulations are capable of producing accurate results at all (X.Q. Sheng, J.-M. Jin, J.M. Song, C.C. Lu, W.C. Chew, *IEEE Trans. AP*, pp. 303–311, Mar. 1998).
In this work, we consider a standard formulation based on the electric field integral equation (EFIE) discretized using a Galerkin procedure involving RWG basis functions on triangular meshes in the BI along with edge elements on tetrahedral meshes in the FE and in previous publications it was shown that the formulation is capable of producing accurate results. However, in the past we solved the linear system of equations by direct LU decomposition techniques restricting applicability of the algorithm to rather small problems. In order to tackle larger problems, the algorithm was extended by an iterative solver in conjunction with the multi-level fast multipole method for the fast evaluation of the BI matrix-vector products. Solving large problems involving irregular FE meshes requires a good preconditioner for the system matrix. Preconditioners are typically generated using a sparse approximation of the system matrix that is factorized by direct LU decompositions. However, during the factorization process new matrix elements (fill-ins) are generated and in order to keep computer resources little, most of these fill-ins must be suppressed. Experience shows that the preconditioner becomes easily useless if too many fill-ins are dropped and consequently the whole procedure may be very inefficient and its behavior is rather unpredictable. To avoid such problems, we employ an iterative preconditioning strategy related to that one proposed by Liu and Jin in (J. Liu and J.-M. Jin, *IEEE Trans. AP*, pp. 1212–1221, Sep. 2002.). In contrast to introducing a further local absorbing boundary condition operator into the formulation, our preconditioner is directly derived from the FE - BI equation system. As usual, we choose a sparse approximation of the FE - BI system and preconditioning operators are generated using a few iterations of the generalized minimal residual (GMRES) algorithm. Overall convergence of the iteration loop is very good and in contrast to incomplete LU decomposition techniques the behavior of the iteration loop can be controlled by adjustment of the iteration count in the preconditioner dependent on the observed convergence. Numerical results will be shown to demonstrate the usefulness of the proposed iterative preconditioning strategy.