A New Adaptive Spatial Smoothing Method

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1. Introduction

Because of mulitpath or smart jamming, the desied signal of an adaptive array will be correlated or coherent (when $|\rho| = 1$) with interferers. Spatial smoothing is a simple and effective technique for solving this problem[1][2][3] and has been widely used for estimation of Direction of Arrival (DOA) and Digital Beamforming(DBF). However, the array aperture is reduced because subarray has to be used. Consequently, the resolution of array is reduced. If we want to maintain large aperture subarray, the number of subarrays is decreased and the effect of decorrelation is degraded which deteriorate the array performance. This paper focuses on the DBF application. An adaptive spatial smoothing method is proposed. Compared with the traditional uniform spatial smoothing method, the advantages of this method are: (1) When the incident angles between interferers and desired signal are small, the array gain is improved. (2) When large subarry aperture is required, the adaptive spatial smoothing method can provide better array gain.

2. Spatial Smoothing Method

We consider a uniform linear array with N elements. The dimension of subarray is M. Thus we can obtain $L = N - M + 1$ subarrays. The distance between adjacent elements is $\lambda/2$,where λ is the wavelength of desired narrow-band signal. Assume D narrow-band incident planewave signals including one desired signal and $D-1$ interferers. One or more interferers are correlated with the desired signal. The frequency domain snapshot model is used, $X(\omega) = VF(\omega) + W(\omega)$, where $F(\omega)$ is a $D \times 1$ random vector with zero mean value, which is defined as $F(\omega) = [s(\omega), I_1(\omega), \cdots, I_{D-1}(\omega)]^T$. $W(\omega)$ is additive white noise with power σ_ω^2. V is the manifold matrix of array which is represented as $V = [v_N(\varphi_1) \vdots v_N(\varphi_2) \vdots \cdots \vdots v_N(\varphi_D)]$,Where $v_N(\varphi) = [1, e^{j\varphi}, e^{j2\varphi}, \cdots, e^{j(N-1)\varphi}]^T$ and $\varphi_i = \pi \sin(\theta_i) \quad i = 1,2,\cdots D$, where θ_1 is the incident direction of the desired signal and

$\theta_i, i = 2, \cdots, D$ are the incident directions of interferers. The covariance matrix of snapshots is $S_x = V\ S_f V^H + \sigma_\omega^2 I$, where S_f is the covariance matrix of $F(\omega)$. We choose an M-dimension subarray and define $V_M = [v_M(\varphi_1) \vdots v_M(\varphi_2) \vdots \cdots \vdots v_M(\varphi_D)]$ and $D = diag[e^{j\varphi_1} \vdots e^{j\varphi_2} \vdots \cdots \vdots e^{j\varphi_D}]$.The covariance matrix of the ith subarray can be written as $S_M^{(i)} = V_M D^{(i-1)} S_f (D^{(i-1)})^H V_M^H + \sigma_\omega^2 I$. The traditional forward smoothing method uniformly weights all subarrays. That is $S_{FS} = \frac{1}{L} \sum_{i=1}^{L} S_M^{(i)}$, which can be further expressed as

$$S_{FS} = V_M [\frac{1}{L} \sum_{i=1}^{L} D^{(i-1)} S_f (D^{(i-1)})^H] V_M^H + \sigma_\omega^2 I = V_M S_{f,FS} V_M^H + \sigma_\omega^2 I$$

We can observe that the effect of spatial smoothing is equivalent to construct an M-element array and the covariance matrix of $F(\omega)$ is changed from S_f to $S_{f,FS}$. The (i,j)th element of $S_{f,FS}$ can be expressed as $S_{f,FS}(i,j) = S_f(i,j) \frac{1}{L} \sum_{i=1}^{L} e^{j(i-1)\Delta\varphi_{ij}}$. Therefore, the effect of spatial smoothing is equivalent to multiply a factor $g_{ij} = \frac{1}{L} \sum_{i=1}^{L} e^{j(i-1)\Delta\varphi_{ij}}$ with the original (i,j)th off-diagonal element of S_f while preserving all original diagonal elements.

3. Adaptive spatial smoothing

Assume that the direction of the desired signal satisfies $\varphi_1 = 0$. We observe that $\Delta\varphi_{1j}$ and $\Delta\varphi_{j1}$ are $-\varphi_j$ and φ_j, $j = 2, \cdots, D$ actually. If we find an L-dimension adaptive weight vector whose correspondent beam pattern has notches at $-\varphi_j$ and φ_j, $j = 2, \cdots, D$, and use this adaptive weight to perform adaptive weighting smoothing, then the correlation of signals can be greatly reduced and the performance of array can be improved when $|\Delta\varphi_{1j}|$ is small. Furthermore, (1) Because the beam pattern of L-dimension linear array is symmetrical about $\varphi_1 = 0$, we use real weight vector and constraint the sum of all elements to be 1. (2) Because here the L-dimension linear array must restrain two times the original number of interferers, we must ensure $L > 2D - 1$.

The steps of adaptive spatial smoothing method are :

<u>Step 1:</u> We choose the dimension of subarray as L and $M = N - L + 1$ subarrays are obtained. Then we use traditional uniform spatial smoothing method (or Chebychev weighting method) to get an $L \times L$ autocorrelation matrix R_L. We take the real part of R_L as

$$R_L^{'} = \mathrm{Re}(R_L) = E[\mathrm{Re}(X_L(\omega))(\mathrm{Re}(X_L(\omega)))^H] - E[\mathrm{Im}(X_L(\omega))(\mathrm{Im}(X_L(\omega)))^H]$$

For convenience, we consider the situation of one desired signal and one correlated interferer and assume that $E[\begin{bmatrix} s(t) \\ I(t) \end{bmatrix}[s(t)^* \quad I(t)^*]] = [\begin{matrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho^*\sigma_1\sigma_2 & \sigma_1^2 \end{matrix}]$. we have

$$R_L^{'} = \frac{1}{2} V_L \begin{bmatrix} \sigma_1^2 & 0 & g_{13}\rho\ \sigma_1\sigma_2 & 0 \\ 0 & \sigma_1^2 & 0 & g_{24}\rho^*\sigma_1\sigma_2 \\ g_{31}\rho^*\sigma_1\sigma_2 & 0 & \sigma_2^2 & \\ 0 & g_{42}\rho\ \sigma_1\sigma_2 & 0 & \sigma_2^2 \end{bmatrix} V_L^H \text{ , where}$$

g_{ij} denotes the multiplication factor introduced in the smoothing process. Therefore, $R_L^{'}$ is an autocorrelation matrix of 4 signals and the manifold matrix is $V_L = [v_L(\varphi_1) \vdots v_L(-\varphi_1) \vdots v_L(\varphi_2) \vdots v_L(-\varphi_2)]$. The result can be extended to the situation of more interferers. We can obtain the adaptive weight vector using MPDR algorithm $W_L = \dfrac{(R_L^{'})^{-1} I_L}{I_L^H (R_L^{'})^{-1} I_L}$, where $I_L = [1,1,\cdots,1]^T, L \times 1$. W_L is a real vector. Due to the effect of spatial smoothing , the beam pattern correspondent to W_L has low values at $-\varphi_j$ and φ_j, $j = 2,\cdots,D$.

Step 2: We perform adaptive spatial smoothing on L M-dimension subarrays with W_L, where $S_{f,FS}(i,j) = S_f(i,j)\sum_{i=1}^{L} W_{L,i}^* e^{j(i-1)\Delta\varphi_{ij}} = \rho_{ij}\sigma_i\sigma_j \sum_{i=1}^{L} W_{L,i}^* e^{j(i-1)\Delta\varphi_{ij}}$.

Step 3: Based on step 2, we use MPDR algorithm again and obtain an M-dimension subarray weight vector, $W_M = \dfrac{(S_{FS})^{-1} I_M}{I_M^H (S_{FS})^{-1} I_M}$, where $I_M = [1,1,\cdots,1]^T$, $M \times 1$.

4. Simulation results

We consider a standard 16-element linear array. The dimension of subarray is ranged from 9 to 13. The direction of the desired signal is $\theta_1 = 0$. There is one interferer whose direction $u = \sin(\theta_2)$ is uniformly distributed between [0.2,1]. The correlation coefficient of the desired signal and interferer is real and uniformly distributed in [0,1].

Additive white noise is assumed. $SNR = 20$ (dB) and $INR = 20$ (dB). The snapshot number of the simulation is 1000.

In Figure 1, we demonstrated the statistical results of array gain for different interferer incident angles. We observe that when the incident angle of interferers is small, adaptive spatial smoothing method is the best one.

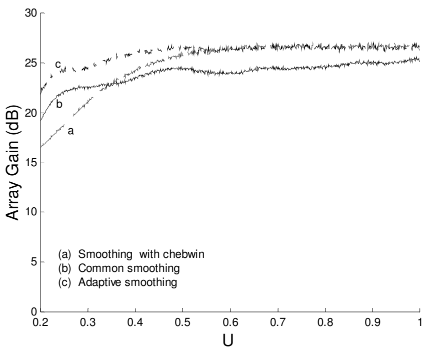

Figure 1 Performance versus incident angles

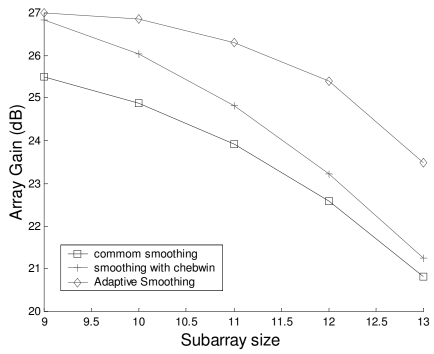

Figure 2 Performance versus subarray apertures

In Figure 2, we show the performance for different subarray dimensions M=9~13. The statistical results of array gain are demostrated. We can observe that when the dimension of subarray M is large, the adaptive spatial smoothing method performs better than other two methods.

5. Conclusion

Compared with the traditional spatial smoothing method and the weighted spatial smoothing method, the new method has following advantages: (1) When the incident angles between interferers and the desired signal are small, the array gain is improved.(2) When a large subarry aperture is required, better array gain can be obtained. In the step 1 of adaptive spatial smoothing , we need to compute an L×1 adaptive weight, which brings additional computational load. But because large M will lead to small L and the computation is based on real number, the computational cost is acceptable.

References

[1] TJ Shan and T.Kailath. Aaptive beamforming for coherent signals and interference. IEEE trans. ASSP, vol. ASSP-33,pp.527-534,June,1985。

[2] V.U.Reddy, A.Paulraj, and T.kailath. Performance analysis of the optimum beamformer in the presence of correlated sources and its behavior under spatial smoothing. IEEE Trans. ASSP,vol.ASSP-35,pp.927-936,July 1987。

[3] C.-J.Tsai, J.F.Yang, and T.H.Shiu. Performance analysis of beamformers using effective SINR on array parameters. IEEE Trans. Signal Process.,vol.SP-43,pp.300-303,January 1995.