The Pre-processing Method Based on Signal Conjugate Cyclostationary

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Abstract: In this paper we propose a kind of pre-processing method of conjugate cyclic correlation. These methods make use of the information of the matrix of conjugate cyclic correlation is Hankel matrix. Under the condition of low signal-noise ratio (SNR), these pre-processing methods can improve the performance of direction-of-arrival (DOA). Its effectiveness is illustrated by the simulation results.

Ⅰ Introduction

Conventional array processing methods basically rely on the spatial properties of the signals impinging on an array of sensors. In practice, using the temporal property of signals can improve the performance of DOA estimation. Recently the cyclostationary, which is signal temporal property, has been widely considered for signal processing. The cyclostationary concept is first introduced into array signal processing by Garner and Schell [1, 2]. Then, the Cyclic MUSIC (CM) and conjugate Cyclic MUSIC (CCM) presented in [1, 2] accommodate multiple signals having the same cycle frequency. The Cyclic ESPRIT (CE) is also reported in [1], the method of spectral correlation signal subspace fitting (SC-SSF) is proposed in [3], and the method of general spectral correlation signal subspace fitting (GSC-SSF) is presented in [4]. In this paper, we proposed some pre-processed methods called Hankel approximation method (HAM) by analysis the cyclic matrix.

Ⅱ Conjugate Cyclostationary

We can obtain the conjugate cyclic correlation of $x_p(t)$ and $x_q(t)$ is defined as in [3]

$$\boldsymbol{R}^{\alpha}_{x_p x_q^*}(\tau) \approx \sum_{n=1}^{N} \boldsymbol{R}^{\alpha}_{s_n s_n^*}(\tau) e^{-j\pi\alpha(\tau_{pn}+\tau_{qn})} e^{-j2\pi f_0(\tau_{pn}+\tau_{qn})} = \sum_{n=1}^{N} \boldsymbol{R}^{\alpha}_{s_n s_n^*}(\tau) e^{-j\pi(2f_0+\alpha)(p+q-2)\tau_n} \quad (1)$$

where the α is cyclic frequency. We apply the lemma 1 of narrow-band assumption in [3] to prove the equation (1). However, if we directly apply the assumption of narrow-band $s_i(t-\tau_{pk}) = s_i(t-(p-1)\tau_k) \approx s_i(t)e^{-j\omega_0(p-1)\tau_k}$, then another the conjugate cyclic correlation function of $x_p(t)$ and $x_q(t)$ can be given by

$$\boldsymbol{R}^{\alpha}_{x_p x_q^*}(\tau) \approx \sum_{n=1}^{N} \boldsymbol{R}^{\alpha}_{s_n s_n^*}(\tau) e^{-j2\pi f_0(p+q-2)\tau_n} \quad (2)$$

We note the equation (1) and equation (2) is unequal. But the two equation is approximate equal if $f_0 >> \alpha$. The difference of the two equations is exponential

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frequency. Therefore, we can conclude the relation of equation (1) and equation (2)

$$\boldsymbol{R}^{\alpha}_{x_p x_q^*}(\tau) \approx \sum_{n=1}^{N} \boldsymbol{R}^{\alpha}_{s_n s_n^*}(\tau) e^{-j2\pi f(p+q-2)\tau_n} = r^{\alpha}(p+q-2,\tau) \tag{3}$$

The equation (3) is equal equation (2) when $f = f_0$, and the equation (3) is equal equation (1) when $f = f_0 + \alpha/2$. Therefore the conjugate cyclic correlation of $x_p(t)$ and $x_q(t)$ is equation (3).

Ⅲ The methods of conjugate cyclic cyclostationary

A new matrix $\boldsymbol{R}_3$ can be restructured, and the p row and the q column of the new matrix is $\boldsymbol{R}^{\alpha}_{x_p x_q^*}(\tau)$. Assuming the frequency $f = f_0$, then we get

$$\mathbf{R}(\alpha) = \mathbf{A}_3^{\alpha} \mathbf{R}^{\alpha}(\alpha) \left(\mathbf{A}_3^{\alpha}\right)^H = \begin{bmatrix} r^{\alpha}(0,\tau) & \cdots & r^{\alpha}(M-2,\tau) & r^{\alpha}(M-1,\tau) \\ r^{\alpha}(1,\tau) & \cdots & r^{\alpha}(M-1,\tau) & r^{\alpha}(M,\tau) \\ \vdots & \ddots & \vdots & \vdots \\ r^{\alpha}(M-1,\tau) & \cdots & r^{\alpha}(2M-3,\tau) & r^{\alpha}(2M-2,\tau) \end{bmatrix} \tag{4}$$

where $\mathbf{R}(\alpha)$ is the matrix combined with signal function of conjugate cyclic correlation, $\boldsymbol{A}_3^{\alpha} = [\boldsymbol{a}_3^{\alpha}(\theta_1), \boldsymbol{a}_3^{\alpha}(\theta_2), \cdots, \boldsymbol{a}_3^{\alpha}(\theta_N)]$ and $\boldsymbol{a}_3^{\alpha}(\theta) = [1, e^{-j2\pi f\tau_n}, \cdots, e^{-j2\pi f(M-1)\tau_n}]^T$.

The above equation shows the matrix $\mathbf{R}(\alpha)$ is Hankel matrix, whether the frequency $f = f_0$ or $f = f_0 + \alpha/2$. Therefore, we can reconstruct a Hankel matrix $\boldsymbol{R}_H$ from $\mathbf{R}(\alpha)$ in the minimum metric distance sense by solving the following problem

$$\min_{R_H \in S_H} |\boldsymbol{R}_H - \boldsymbol{R}(\alpha)| \tag{5}$$

where S_H is the set of Hankel matrices. Now we introduce three Hankel approximation methods.

1. Hankel approximation method (HAM)

The modified Hankel matrix $\boldsymbol{R}_H$ has the entries given as follows

$$\hat{r}_H(n) = \frac{1}{n+1} \sum_{i=1}^{n+1} \hat{r}_{i(n+2-i)}, 0 \le n \le M, \quad \hat{r}_H(n) = \frac{1}{2M-n-1} \sum_{i=n-M+2}^{M} \hat{r}_{i(n+2-i)}, M < n \le 2M-2 \tag{6}$$

where M is the number of sensor, $\hat{r}_{ij}$ is element of $\boldsymbol{R}(\alpha)$, and $\hat{r}_{Hij} = \hat{r}_H(|i-j|)$.

2. Modified Hankel approximation method (MHAM)

From above equation, we know the essential thought of HAM is the average of anti-diagonal elements of $\boldsymbol{R}(\alpha)$. The new modified method will get if the elements of $\boldsymbol{R}_H$ are replaced by

$$\hat{r}_H(n) = \rho(n) e^{j\varphi(n)} \tag{7}$$

where $\rho(n)$ represents the amplitude of $\hat{r}_H(n)$ and $\varphi(n)$ represents the phase of

$\hat{r}_H(n)$, and

$$\hat{\varphi}_H(n)=\frac{1}{n+1}\sum_{i=1}^{n+1}\hat{\varphi}_{i(n+2-i)},0\le n\le M\,,\hat{\varphi}_H(n)=\frac{1}{2M-n-1}\sum_{i=n-M+2}^{M}\hat{\varphi}_{i(n+2-i)},M<n\le 2M-2 \quad (8)$$

Therefore, other modified method will get by

$$\rho_H(n)=\frac{1}{n+1}\sum_{i=1}^{n+1}\rho_{i(n+2-i)},0\le n\le M\,,\rho_H(n)=\frac{1}{2M-n-1}\sum_{i=n-M+2}^{M}\rho_{i(n+2-i)},M<n\le 2M-2 \quad (9)$$

Ⅳ Simulation Result

In this section, two computer simulation examples are presented for showing the effectiveness of the proposed methods. We compare the conjugate cyclostationary method (CCM) and HAM pre-processing methods (HAM and MHAM). To examine the effectiveness of the modified HAM and MHAM methods are presented for 8-sensors uniform linear array. Two sources of same cyclic frequency were equally powered and their bearings were fixed at 20° and 30°, respectively. One hundred independent runs are performed for every experiment. Fig 1 shows the relation of estimation performance with independent sources and SNR. Fig 1(a) shows the probability of estimation. Fig 1(b) shows the relation of estimation error and SNR. Fig 1(c) shows the relation of estimation variance and SNR. Fig 2 shows the correlation sources estimation performance. The coefficient of correlation is 0.5. Fig 2(a) is the relation of estimation error and SNR. Fig 2(b) is the relation of estimation variance and SNR.

Ⅴ. Conclusion

In this paper, we propose the HAM pre-processing methods based on analysis conjugate cyclostationary matrix. Under the condition of low SNR, these methods can improved the performance of DOA estimation and the probability of estimation. The simulation illustrate these pre-processing algorithms have robust performance.

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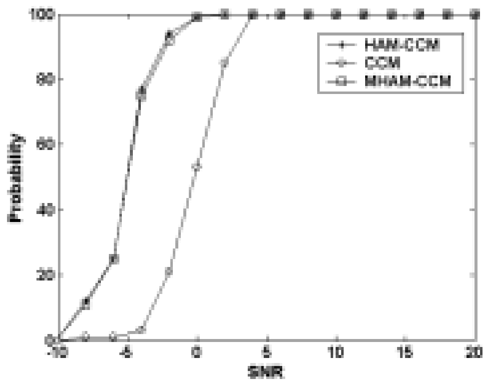

Figure 1(a)

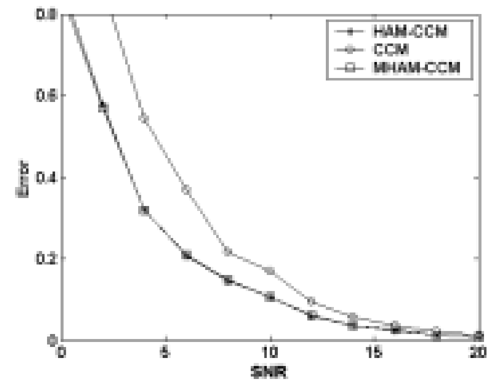

Figure 1(b)

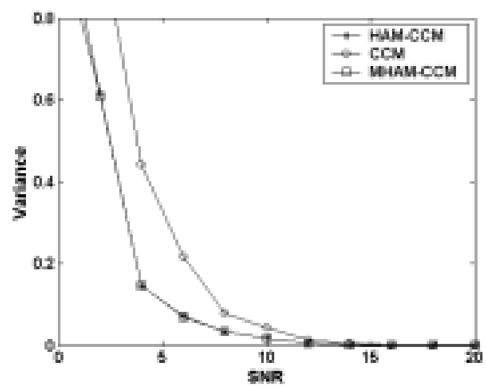

Figure 1(c)

Fig.1 the performance of Hankel pre-processing method , Source independent

(a) Probability (b) Error (c) Variance

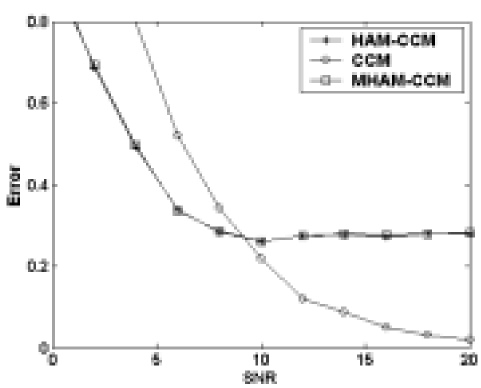

Figure 2(a)

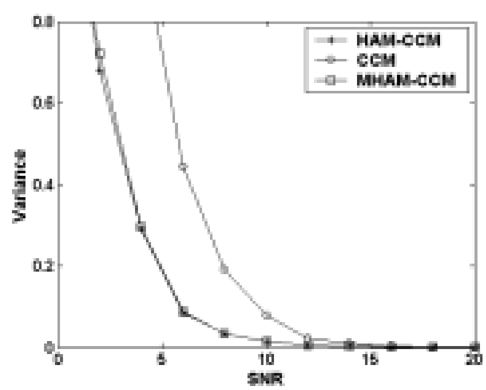

Figure 2(b)

Fig.2 the performance of Hankel pre-processing, the coefficient of correlation is 0.5

(a) Estimation Error (b) Estimation Variance