Radio Wave Propagation Over a Rectangular Obstacle

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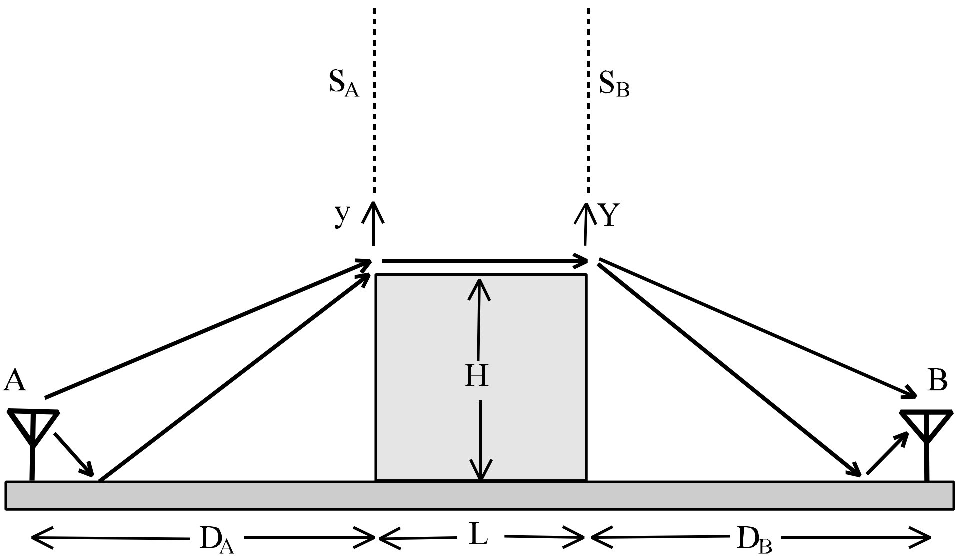
1. Introduction

With the increased use of radio communications systems in cluttered urban environments, there is a growing need for techniques that can be used to estimate the strength of non line of sight propagation. An important model problem for such calculations, and that to which this paper is addressed, concerns the propagation of radio waves over a rectangular dielectric body. In the case of PEC materials, this problem has been considered by Lee [1],



Whitteker [2], Elides [3] and Ong and Constantinou [4] using what is essentially a discretised version of Feynman path integration. This approach can be reinterpreted as the repeated application of an integral equation and this alternative procedure has been suggested as a technique for studying propagation problems in general [5,6]. The region between transmitter and receiver is dissected into slices by means of suitable surfaces and the solution progressed between these surfaces by means of suitable integral equations. Using only a single intermediate surface, Monteath [7] has shown that such an approach can be effective for several non trivial problems. For propagation that does not deviate too greatly from line of sight, the integral equation can be simplified to

$$\underline{E}(\underline{r}_0).\underline{J}_0 = -2\int_S \frac{\underline{E}.\underline{E}^0}{\eta}\, dS \qquad \textbf{(1)}$$

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which relates the electric field at point $\underline{r}_0$ to its behaviour on a surface S that separates the field sources from the point $\underline{r}_0$ (electromagnetic field $\underline{E}^0$ is the electric field resulting from a Hertzian dipole $\underline{J}_0$ located at point $\underline{r}_0$). The surface S should be perpendicular to the line of sight and, as a minimum, there should be surfaces at major features such as corners and changes in material. Importantly, it should be noted that the medium between the surfaces can be non homogenous in nature, providing that dipole solution $\underline{E}_0$ is a valid in such a medium. As a consequence, the approach can be used to investigate radio wave propagation over a non PEC rectangular obstacle. It is the purpose the current paper to use the integral equation approach to derive a simple approximate expression, valid in the high frequency limit, for calculating the strength of such propagation. The approach can also applied to the case of propagation over a PEC obstacle and the resulting expression provides values of attenuation that are within a few dB of the theoretical and experimental results of reference [4].

2. Derivation of the Propagation Formula

It is assumed that the wavelength is very much shorter than all other length scales and so a geometric optics dipole approximation will be appropriate for the integral equations. Furthermore, it is assumed that the antennas lie in each others shadow region and so the major communication between these devices will be through energy that is diffracted around the corners. We will use equation 1, evaluated on surface S_A, to calculate the field of antenna A on surface S_B. The dominant contribution to the surface integrals will come from the vicinity of the plateau where the vertical component of electric field is dominant. Consequently, for a unit vertical dipole ($\underline{J}_0 = \hat{\underline{j}}$) in surface S_B,

$$\begin{aligned}\underline{E}^0 \approx \frac{j\omega\mu}{4\pi L}\left(\exp\left(\frac{j\beta yY}{L}\right) + R_V \exp\left(-\frac{j\beta yY}{L}\right)\right) \times \\ \times \exp\left(-j\beta\left(L + \frac{(x-X)^2 + y^2 + Y^2}{2L}\right)\right)\hat{\underline{j}}\end{aligned} \tag{2}$$

with reflection coefficient R_V approximated by $R_V \approx -1 + 2\dfrac{y+Y}{L\eta_r}$.

Furthermore, the field due to antenna B can be approximated by

$$\underline{E}^B \approx \frac{j\omega\mu_0 I_B}{4\pi}\underline{h}^B \frac{\exp(-j\beta R_B)}{R_B}\left(1 - \frac{HY}{R_B^2}\right)\exp\left(-j\beta\frac{Y^2 + 2HY + X^2}{2R_B}\right) \tag{3}$$

where $R_B = \sqrt{H^2 + D_B^2}$ and $\underline{h}^B$ is the effective length of antenna B ($\underline{h}^B$ includes effects such as ground reflection and the height of the antenna above the ground). From equation 1, the field on surface S_A is given by

$$E_y = \frac{\omega^2 \mu_0^2 I_B h_y^B}{8\pi^2 \eta_0} \frac{\exp\left(-j\beta\left(R_B + L\right)\right)}{R_B L} \int_0^{\infty} \int_{-\infty}^{\infty} \left(\exp\left(\frac{j\beta y Y}{L}\right) + R_V \exp\left(-\frac{j\beta y Y}{L}\right)\right) \times$$
$$\times \left(1 - \frac{Hy}{R_B}\right) \exp\left(-j\beta\left(\frac{Y^2 + 2HY + X^2}{2R_B} + \frac{(x - X)^2 + y^2 + Y^2}{2L}\right)\right) dX\, dY \qquad (4)$$

For small y, and in the limit $\omega \to \infty$, the above expression yields

$$E_y \approx \frac{\omega^2 \mu_0^2 I_B h_y^B}{8\pi^2 \eta_0} \sqrt{\frac{2\pi}{j\beta R_B L\left(R_B + L\right)}} \times$$
$$\times \left(\frac{2yR_B^2}{j\beta H^2 L} - \frac{2R_B^2}{\beta^2 \eta_r H^2 L} + \frac{2yR_B}{j\beta\eta_r HL}\right) \exp\left(-j\beta\left(R_B + L + \frac{1}{2}\frac{x^2}{L + R_B} + \frac{y^2}{2L}\right)\right) \qquad (5)$$

An alternative form of equation (1) is given by

$$VI = 2\int_S \frac{\underline{E}.\underline{E}^0}{\eta}\, dS \qquad (6)$$

where V is the voltage induced in an antenna by field $\underline{E}$ ($\underline{E}^0$ is the field caused by current I in the antenna). The field due to antenna A is given by

$$\underline{E}^A \approx \frac{j\omega\mu_0 I_A}{4\pi} \underline{h}^A \frac{\exp\left(-j\beta R_A\right)}{R_A} \left(1 - \frac{HY}{R_A^2}\right) \exp\left(-j\beta \frac{y^2 + 2Hy + x^2}{2R_A}\right) \qquad (7)$$

where $\underline{h}^A$ is the effective length of antenna A. Applying equation (6) on surface S_A, we obtain

$$V_{AB} \approx \frac{j\omega\mu_0 I_B h_y^B h_y^A}{4\pi^2} \sqrt{\frac{R_A R_B}{L\left(R_A + R_B + L\right)}} \frac{1}{\beta^2 H^3 L} \times$$
$$\times \left(\frac{R_A R_B}{H} + \frac{R_A}{\eta_r} + \frac{R_B}{\eta_r}\right) \exp\left(-j\beta\left(R_B + L + R_A\right)\right) \qquad (8)$$

where V_{AB} is the voltage induced in antenna A by current I_B in antenna B. The above approximations can also be applied in the case of a PEC obstacle, but in this case we will have $R_V = 1$ and equation 9 will be replaced by

$$V_{AB} \approx -\frac{\omega\mu_0 I_B h_y^B h_y^A}{4\beta\pi^2 H^2}\sqrt{\frac{R_A R_B}{L\left(R_A + R_B + L\right)}}\exp\left(-\ j\beta\left(R_B + L + R_A\right)\right) \qquad (9)$$

In the case of an object that is perfectly absorbing, the voltage in antenna A will be one half the value above.

3. Discussion

We have derived a simple expression for propagation over a rectangular dielectric obstacle, but need to consider the range of parameters for which this expression is valid. Firstly, the requirement that the paths do not deviate too far from line of sight will dictate that both R_A and R_B be at least $2H$. Furthermore, the approximations used in evaluating the integrals will require that both R_A and R_B be no more than $2H\sqrt{L/\lambda}$ in magnitude (we have implicitly assumed that L is much less than both R_A and R_B). Although the permissible values of both R_A and R_B are limited, they nevertheless cover a range that is extremely useful in practical terms. In the case of a PEC obstacle, equation (9) yields results that are within a few dB of the theoretical and experimental results given in reference [4]. References [2] and [3] have studied propagation over a PEC obstacle with sloping roof and it is a simple matter to extend the above considerations to the study of propagation over a dielectric obstacle with a sloping roof.

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