A Method for Wideband Direction-of-Arrival Estimation Using Frequency-Domain Frequency-Invariant Beamformers

Tuan Do-Hong*, Franz Demmel**, Peter Russer*

*Munich University of Technology.
Arcisstrasse 21, 80333 Munich, Germany.
E-mail: do-hong@ei.tum.de

**Rohde & Schwarz Company.
Muehldorfstrasse 15, 81671 Munich, Germany.

Abstract

In this paper, we propose a new wideband direction-of-arrival (DOA) estimation method for wideband smart antennas based on Frequency-Domain Frequency-Invariant Beamformers (FDFIB). By appropriately designing the weights for frequency-domain beamformers at different frequencies, the frequency-invariant beam-patterns are obtained. These beamformers play a role as (wideband) spatial processor transforming signals from element-space to beam-space. After that, the narrowband DOA methods can be applied in beam-space to estimate DOAs. The method is able to operate with arbitrary antenna array and arbitrary wide bandwidth without preliminary DOA estimate. In addition, the proposed method provides low computational time and simple implementation. The performance of proposed method in comparison with conventional wideband methods is illustrated in simulation results.

I. Introduction

The smart antennas are one of the best solution for increasing the system capacity and performance in future mobile communication systems. Two major issues in smart antennas are direction-of-arrival (DOA) estimation and beamforming. Moreover, in future communication systems, wideband signals will be used to fulfill the requirements of higher data rate. However, most of proposed methods for these issues in literature are with narrowband assumptions. The narrowband signal is assumed to have the ratio of bandwidth to center frequency (fractional bandwidth) less than 1%. Wideband and ultra-wideband applications are proposed for fractional bandwidth up to $50 - 200\%$ [1].

Initial wideband DOA estimation method introduced in [3] for incoherent signal sources. The Coherent Signal-Subspace (CSS) [4] is an alternative method to deal with coherent sources. In this method, the wideband array data are first decomposed into several narrowband components via fast Fourier transform (FFT). Focusing matrices are calculated to transform each of the narrowband array manifold matrices into the one corresponding to a selected frequency. The narrowband DOA estimation method [2] is then applied to find the DOAs. The design of focusing matrices in CSS method requires the preliminary DOA estimates in the neighborhoods of the true DOAs. The CSS method is further developed in [5]-[7]. In the beam-space-CSS method proposed in [8], the concept of beam-space manifold invariance is introduced. A design criterion based on least squares fit is employed to construct a beamforming matrix for each frequency. Recently, a method proposed in [9] performs beam-space processing using time-domain frequency-invariant beamformers (TDFIB). The method does not require preliminary DOA estimates and has less computational complexity than beam-space-CSS in [8]. However, the frequency-invariant characteristic depends on the number of antenna elements within the arrays, on the geometry of arrays and on the design of filters in TDFIB.

In this paper, we propose a new method for wideband DOA estimation using arbitrary antenna array based on *Frequency-Domain Frequency-Invariant Beamformers* (FDFIBs). It is different from time-domain beamformer method in [9], and from the method in [8] where frequency-domain beamformers (FDBs) are designed by solving a multi-parameter least squares optimization problem (computation complexity); instead, we use FDBs with appropriately designed weights at different frequencies to ensure that beam-patterns of FDBs are constant over the frequency band, i.e. frequency-invariant beamformer. In this method, the FDFIBs play a role as spatial processors transforming the element-space into the beam-space, the narrowband method such as MUSIC [10] is then applied in beam-space at single selected frequency within the bandwidth to estimate DOAs.

0-7803-7846-6/03/$17.00 ©2003 IEEE

II. Frequency-Domain Frequency-Invariant Beamformer (FDFIB)

Consider an arbitrary array composed of M identical elements, all elements are assumed to be omnidirectional and no mutual coupling between elements. A beamforming network, which consists of J frequency-domain beamformers, is considered. Wideband incoming signals are considered in the frequency range from ω_l to ω_h, where ω_l and ω_h are lowest and highest frequencies, respectively. The j-th beam-pattern, $B_j(\omega_k), j = 1...J$, at frequency ω_k, $\omega_l \le \omega_k \le \omega_h$, is given by

$$B_j(\omega_k) = \mathbf{w}_j^H(\omega_k)\mathbf{b}(\omega_k, \mathbf{\Omega}_b) \tag{1}$$

where $\mathbf{b}(\omega_k, \mathbf{\Omega}_b) = [e^{-j\frac{\omega_k}{c}\mathbf{d}_1^T\mathbf{\Omega}_b}, \ldots, e^{-j\frac{\omega_k}{c}\mathbf{d}_m^T\mathbf{\Omega}_b}, \ldots, e^{-j\frac{\omega_k}{c}\mathbf{d}_M^T\mathbf{\Omega}_b}]^T$ is steering vector at b-th direction, $\mathbf{d}_m = [\mathrm{d}_{x_m}, \mathrm{d}_{y_m}, \mathrm{d}_{z_m}]^T$ denotes the co-ordinates of the m-th element within the array, $m = 1...M$; $\mathbf{\Omega}_b = [\cos\phi_b \sin\theta_b, \sin\phi_b \sin\theta_b, \cos\theta_b]^T$, where ϕ_b and θ_b are azimuth and elevation at b-th direction, respectively, $-\pi \le \phi_b \le \pi,\ 0 \le \theta_b \le \frac{\pi}{2}$, and c is propagation speed. In (1), the superscript H indicates conjugate transpose, and $\mathbf{w}_j(\omega_k)$ is the weighting vector of j-th beamformer, $\mathbf{w}_j(\omega_k) = [w_{j1}(\omega_k), \ldots, w_{jm}(\omega_k), \ldots, w_{jM}(\omega_k)]^T$, where $w_{jm}(\omega_k)$ is the weight at frequency ω_k of j-th beamformer at m-th antenna element, as depicted in Fig. 1.

To obtain frequency-invariant beamformer, the weights at each frequency are chosen such that

$$B_j(\omega_k) = B_j(\omega_0),\ \forall\omega_k \in [\omega_l, \omega_h] \tag{2}$$

where ω_0 is focusing frequency selected in $[\omega_l, \omega_h]$. From (1) and (2), the weights for frequency-invariant beamformer at each frequency are determined as

$$w_{jm}(\omega_k) = \alpha_m e^{-j\frac{\omega_k}{c}\mathbf{d}_m^T\mathbf{\Omega}_b + j\frac{\omega_0}{c}\mathbf{d}_m^T\mathbf{\Omega}_b} \tag{3}$$

where α_m is amplitude weighting coefficients. In this paper, we use $\alpha_m = 1$, $m = 1...M$.

III. Wideband Signal Model in Element- and Beam-Space

Assumed that P wideband signal sources, $s_1(t)...s_P(t)$, with identical bandwidth B, located in the far-field of the array, impinge on the array from distinct directions (ϕ_p, θ_p), $p = 1...P$, where ϕ_p and θ_p are azimuth and elevation, respectively, $-\pi \le \phi_p \le \pi,\ 0 \le \theta_p \le \frac{\pi}{2}$.

Let us denote $\mathbf{x}(n) = [x_1(n), \ldots, x_M(n)]^T$ is vector of signals at output of array. In frequency-domain, we obtain the $(M \times 1)$-vector of signals at outputs of the array as [3]

$$\mathbf{X}(\omega_k) = \sum_{p=1}^{P} \mathbf{a}(\omega_k, \mathbf{\Gamma}_p) S_p(\omega_k) + \mathbf{N}(\omega_k) \tag{4}$$

where $\mathbf{N}(\omega_k)$ is $(M \times 1)$-vector of zero-mean noise at array; $S_p(\omega_k)$ is the Fourier coefficient of the p-th signal at inputs of the array. For the general case of three-dimensional (3-D) arrays, the $(M \times 1)$-source-direction vector of the p-th signal source at frequency bin ω_k is given by

$$\mathbf{a}(\omega_k, \mathbf{\Gamma}_p) = \left[e^{-j\frac{\omega_k}{c}\mathbf{d}_1^T\mathbf{\Gamma}_p}, \ldots, e^{-j\frac{\omega_k}{c}\mathbf{d}_m^T\mathbf{\Gamma}_p}, \ldots, e^{-j\frac{\omega_k}{c}\mathbf{d}_M^T\mathbf{\Gamma}_p}\right]^T \tag{5}$$

where $\mathbf{\Gamma}_p = [\cos\phi_p \sin\theta_p, \sin\phi_p \sin\theta_p, \cos\theta_p]^T$. In matrix notation, equation (4) becomes

$$\mathbf{X}(\omega_k) = \mathbf{A}(\omega_k)\, \mathbf{S}(\omega_k) + \mathbf{N}(\omega_k) \tag{6}$$

where $\mathbf{A}(\omega_k) = [\mathbf{a}(\omega_k, \mathbf{\Gamma}_1), \ldots, \mathbf{a}(\omega_k, \mathbf{\Gamma}_p), \ldots \mathbf{a}(\omega_k, \mathbf{\Gamma}_P)]$ is the $(M \times P)$-source-direction matrix and $\mathbf{S}(\omega_k)$ is the $(P \times 1)$-vector of signals at inputs of the array.

Assumed that P sources are located in a known sector $[\mathbf{\Phi}_l, \mathbf{\Phi}_h]$, and the J-beam beamforming network is designed to cover this sector ($P \le J \le M$). The J signals at outputs of the beamforming network in frequency-domain can be written as

$$\mathbf{Y}(\omega_k) = [Y_1(\omega_k), ..., Y_j(\omega_k), ..., Y_J(\omega_k)]^T = \mathbf{C}^H(\omega_k)\mathbf{X}(\omega_k) = \mathbf{A}_C(\omega_k)\, \mathbf{S}(\omega_k) + \mathbf{N}_C(\omega_k) \tag{7}$$

where $Y_j(\omega_k)$ is Fourier coefficient of beam-space signal, $\mathbf{A}_C(\omega_k) = \mathbf{C}^H(\omega_k)\mathbf{A}(\omega_k)$ is source-direction matrix in beam-space, $\mathbf{N}_C(\omega_k) = \mathbf{C}^H(\omega_k)\mathbf{N}(\omega_k)$ is vector of zero-mean noise in beam-space, $\mathbf{C}(\omega_k) = [\mathbf{w}_1(\omega_k), \ldots, \mathbf{w}_j(\omega_k), \ldots, \mathbf{w}_J(\omega_k)]$ is called as beamforming matrix, and $\mathbf{w}_j(\omega_k)$ is weighting vector of j-th frequency-invariant beamformer defined in section II.

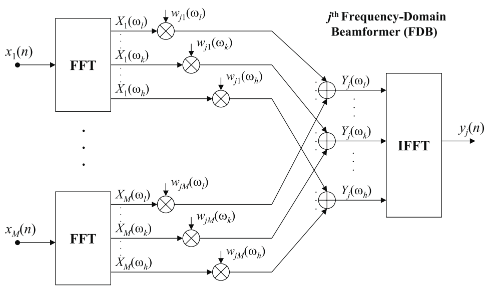

Fig. 1. j-th frequency-domain beamformer.

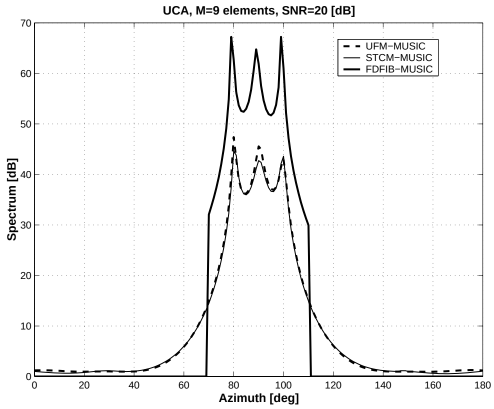

Fig. 2. Estimated spectra at $80°, 90°, 100°$, SNR=20 dB.

IV. Wideband DOA Estimation Using FDFIBs

Because the beamformers are designed to be frequency-invariant, the source-direction matrix in beam-space is constant for all frequencies within the signal bandwidth, i.e. $\mathbf{A}_C(\omega_k) = \mathbf{A}_C(\omega_0)$, $\forall\omega_k \in [\omega_l, \omega_h]$. Therefore, wideband directions-of-arrival are completely characterized by a single source-direction matrix $\mathbf{A}_C(\omega_0)$. Assumed that the signals and the noise are uncorrelated, the cross-spectral density matrix in beam-space is given by

$$\mathbf{R}_Y(\omega_k) = E\{\mathbf{Y}(\omega_k)\mathbf{Y}^H(\omega_k)\} = \mathbf{C}^H(\omega_k)\mathbf{R}_X(\omega_k)\mathbf{C}(\omega_k) \quad (8)$$

where $\mathbf{R}_X(\omega_k) = E\{\mathbf{X}(\omega_k)\mathbf{X}^H(\omega_k)\}$ is the cross-spectral density matrix in element-space. Therefore, the wideband covariance matrix in beam-space can now be obtained as

$$\mathbf{R}_Y = \sum_{k=l}^{h} \mathbf{R}_Y(\omega_k) = \sum_{k=l}^{h} \mathbf{C}^H(\omega_k)\mathbf{R}_X(\omega_k)\mathbf{C}(\omega_k) \quad (9)$$

Define a dense grid of I angle points (or spatial frequency points) of azimuth and elevation. For the MUSIC algorithm [10], the DOAs are determined by searching the peak positions of the following estimated spatial spectrum

$$\hat{S}_{FDFIB-MUSIC}(\mathbf{\Gamma}_i) = \frac{\mathbf{a}_C^H(\mathbf{\Gamma}_i)\mathbf{a}_C(\mathbf{\Gamma}_i)}{\mathbf{a}_C^H(\mathbf{\Gamma}_i)\mathbf{U}_N\mathbf{U}_N^H\mathbf{a}_C(\mathbf{\Gamma}_i)} \quad (10)$$

where $\mathbf{U}_N$ is $(J \times (J-P))$-matrix of noise-eigenvectors obtained from the eigendecomposition of the covariance matrix $\mathbf{R}_Y$, and $\mathbf{a}_C(\mathbf{\Gamma}_i)$ is $(J \times 1)$-source-direction vector in beam-space, $j = 1...I$.

V. Simulation Results

In the simulations, we consider azimuth estimation using an uniform circular array of 9 elements. The co-ordinates of the elements are normalized over $\lambda = \frac{c}{f_h}$, and the radius of array is $r = \lambda$. We compare the performance of proposed method (FDFIB) with UFM [5] and STCM [6] methods. The MUSIC is used in conjunction with these methods. Three uncorrelated wideband sources with normalized frequencies $[\frac{f_l}{f_h}, ..., \frac{f_h}{f_h}] = [0.75, ..., 1]$ (fractional bandwidth of 28.6%) using 201 frequency bins are considered. Assumed that these sources with azimuths $80°, 90°, 100°$ are located in known sector $[70°, 110°]$ and the beamformers are designed to cover this sector using $J = 5$ beams.

The focusing frequency ω_0 is selected at ω_h. The simulations are performed with 500 independent trials and $N = 200$ snapshots are used for each DOA estimate.

The estimated spectra of proposed method (FDFIB-MUSIC), UFM-MUSIC and STCM-MUSIC methods are depicted in Fig. 2. The root-mean-square error (RMSE) of DOA estimates for different values of SNR are shown in Fig. 3. The probability of resolving for various SNR values is shown in Fig. 4. As expected, the FDFIB method exhibits better performance than UFM and STCM methods.

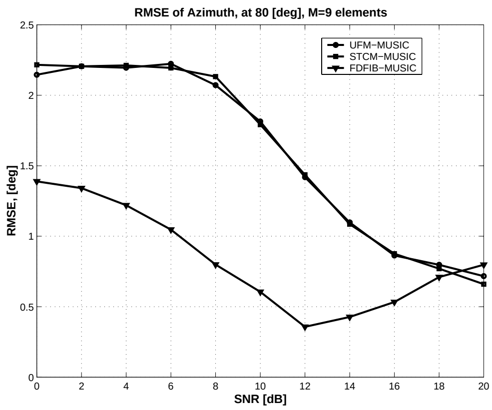

Fig. 3. RMSE vs. SNR at DOA of $80°$.

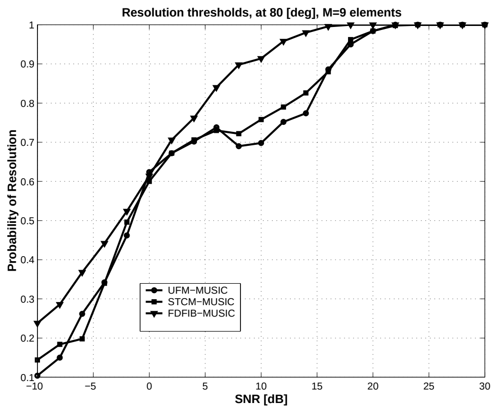

Fig. 4. Probability of resolution vs. SNR.

VI. Conclusion

In this paper, we proposed the new wideband DOA estimation method without preliminary DOA estimation using arbitrary antenna array. The method provides lower resolution threshold and lower RMSE of estimate. Furthermore, due to the parallel processing in frequency-domain beamformer and simple algorithm, the method has low computational time and is suitable for real-time wideband DOA estimation. Unlike the method in [9], where the time-domain beamformers with filters are used, the proposed method can work with low number of element within arbitrary array geometry and does not need additional well-designed filters. The wideband DOA estimation and wideband beamforming methods proposed in this paper are suitable for future wideband smart antennas.

Acknowledgment

The authors wish to acknowledge the support of the Rohde & Schwarz Company, Germany.

References

[1] M. Ghavami, ”Wideband smart antennas theory using rectangular array structures”, *IEEE Trans. Signal Processing*, vol. 50, no. 9, pp. 2143-2151, September 2002.

[2] H. Krim, M. Viberg, ”Two decades of array signal processing research”, *IEEE Signal Processing Marazine*, pp. 67-94, July 1996.

[3] M. Max, T. Shan, T. Kailath, ”Spatio-temporal spectral analysis by eigenstructure methods”, *IEEE Trans. Acoust. Speech, Signal Processing*, pp. 817-827, August 1984.

[4] H. Wang, M. Kaveh, ”Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wide-band sources”, *IEEE Trans. Acoust., Speech, Signal Processing*, pp. 823-831, Aug. 1985.

[5] H. Hung, M. Kaveh, ”Focussing matrices for coherent signal-subspace processing”, *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, no. 8, pp. 1272-1281, August 1988.

[6] J. Krolik, D. N. Swingler, ”Multiple broadband source location using steered covariance matrices”, *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 1481-1494, October 1989.

[7] S. Valaee, P. Kabal, ”Wideband array processing using a two-sided correlation transformation”, *IEEE Trans. Signal Processing*, vol. 43, no. 1, pp. 160-172, January 1995.

[8] T. Lee, ”Efficient wideband source localization using beamforming invariance technique”, *IEEE Trans. Signal Processing*, pp. 1376-1387, June 1994.

[9] D. B. Ward, Z. Ding, R. A. Kennedy, ”Broadband DOA estimation using frequency invariant beamforming”, *IEEE Trans. Signal Processing*, pp. 1463-1469, May 1998.

[10] R. O. Schmidt, ”Multiple emmitter location and signal parameter estimation”, *IEEE Trans. Antennas Propagation*, vol. 34, no. 3, pp. 276-380, March 1986.