Three Discretizations of the MFIE for the Linear Dipole

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1. Introduction

Procedures for determining the currents and input impedance of a linear dipole antenna are generally based on studies of the electric-field integral equation (EFIE), specifically the Pocklington or Hallén equations. Approaches based on the magnetic field integral equation (MFIE) have received little attention other than the work of Zhang and his colleagues in the 1980s [1-3]. Zhang's MFIE implementation employed a series feed and used a low-order method of moments discretization. In the following, the MFIE with a frill feed model is considered. Three discretization procedures, the method of moments (MoM), the boundary residual method (BRM) [4-5], and the locally-corrected Nyström (LCN) method [6-7], are investigated. Since the MFIE is only rigorously applicable to closed conducting bodies, currents on flat end caps are included in the model.

2. Formulation

Consider a linear dipole, of radius a, aligned with the z-axis in a cylindrical coordinate system. By assumption, the fields and current density are ϕ-symmetric. The MFIE for the linear dipole is based on the general relation

$$H_\phi^{inc} = H_\phi^{tot} - H_\phi^{s} \tag{1}$$

which will be enforced in the limit as the observer approaches the surface of the cylindrical dipole from the exterior. The primary unknown on the cylindrical part of the dipole can be expressed in terms of the total current $I(z)$ through the equation

$$I(z) = 2\pi a\, J_z(a, \phi, z) \tag{2}$$

The scattered field due to currents on the barrel can be expressed as

$$H_\phi^{s}(\rho,0,z) = \frac{1}{2\pi}\int_z \int_\phi I(z') \frac{e^{-jkR}}{4\pi R^3}(1 + jkR)(\rho - a\cos\phi')\,d\phi' dz' \tag{3}$$

where

$$R = \sqrt{\rho^2 - 2\rho a\cos\phi' + a^2 + (z - z')^2} \tag{4}$$

The scattered H-field due to a ρ-directed current density on an end cap of the dipole, expressed as $I(\rho) = 2\pi\rho\ J_\rho(\rho, \phi, z')$, can be written as

$$H_\phi^s(\rho,0,z) = -\frac{1}{2\pi}\int_\rho \int_\phi \cos(\phi')I(\rho')\frac{e^{-jkR}}{4\pi R^3}(1+jkR)(z-z')d\rho' d\phi' \qquad (5)$$

where the end cap is located at z' and where

$$R = \sqrt{\rho^2 - 2\rho\rho'\cos\phi' + (\rho')^2 + (z-z')^2} \qquad (6)$$

The incident field is that produced by a magnetic frill of outer radius b located at the center of the dipole, and can be expressed as

$$H_\phi^{inc}(\rho,0,z) = \frac{j\omega\varepsilon}{\ln(b/a)}\int_{\rho'=a}^{b}\int_{\phi'=0}^{2\pi}\cos(\phi')\frac{e^{-jkR}}{4\pi R}d\phi' d\rho' \qquad (7)$$

where R is defined in Equation 6. These integrals can be computed by quadrature with appropriate treatment of the singularities at $R = 0$.

3. Discretization Methods

The MoM procedure is well-known to the EM community and needs no elaboration. A discretization of the MFIE incorporating piecewise-linear basis functions was implemented, with the equation enforced at discrete points at cell junctions to produce a square system of equations.

The BRM, as described in [4-5], was also applied to the MFIE with piecewise-linear basis functions. In this BRM implementation, the equation is enforced at twice the number of boundary points as there are cells in the model, to yield an overdetermined system of equations. In an attempt to better model the current and charge density at the corners where the barrel of the dipole meets the endcaps, the basis functions adjacent to the function at the corner were modified to incorporate a functional dependence

$$B(u) = u^{2/3} \qquad (8)$$

in the cells abutting a corner, where u is a parametric variable along either z or ρ with an origin at the corner. Equation (8) is used instead of the linear function in order to provide the appropriate charge singularity. These functions are superimposed with the regular linear basis functions centered at the corners, so the current at the corner is nonzero.

The LCN procedure [6-7] was implemented with Gauss-Legendre rules of order p up to 10. Order p=2 is expected to provide a representation similar to that of the linear basis functions used with the MoM and BRM [7]. Local corrections were performed in the source cell and the adjacent cells. No attempt was made to incorporate the edge condition as in Equation (8). The LCN procedure does not impose cell-cell continuity and thus uses more unknowns for a given number of cells than the MoM or BRM approaches.

4. Results

For illustration, input impedance results are shown in Tables 1-5 for a dipole of length 0.48λ and radius a=0.0391λ, for a frill feed corresponding to a coaxial ratio b/a=1.187. For this dipole, Holly measured an equivalent impedance of Z_{in} = 102 – j 16 Ω [8]. Numerical results for impedance from the three methods exhibit little variation with changes in the number of cells in the model and agree to within about 1 Ω with each other. The low-order LCN method (*p*=2) provides a representation comparable to the linear basis functions used with the MoM and BRM methods. Higher-order LCN results are shown in Tables 4 and 5, and show only a slight variation from the *p*=2 results. The total number of equations and unknowns are also shown for each approach.

5. References

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TABLE 1. MoM results for Z_{in} of a dipole with L = 0.48λ, a = 0.0391λ, b/a = 1.187

cells on barrel	cells on each end	unknowns	equations	R_{in}	X_{in}
26	1	27	27	83.32	–11.63
50	2	53	53	92.00	–13.59
74	3	79	79	94.29	–14.11
98	4	105	105	95.23	–14.31
124	5	133	133	95.73	–14.41
148	6	159	159	96.00	–14.47
172	7	185	185	96.18	–14.50
196	8	211	211	96.30	–14.52
222	9	239	239	96.39	–14.54
246	10	265	265	96.45	–14.55

TABLE 2. BRM results for a dipole with L = 0.48λ, a = 0.0391λ, b/a = 1.187					
cells on barrel	cells on each end	unknowns	equations	R_{in}	X_{in}
26	1	27	56	104.52	–11.85
50	2	53	108	98.24	–13.87
74	3	79	160	95.97	–13.79
98	4	105	212	95.90	–14.34
124	5	133	268	95.82	–14.57
148	6	159	320	95.76	–14.65
172	7	185	372	95.82	–14.74
196	8	211	424	95.84	–14.77
222	9	239	480	95.87	–14.81
246	10	265	532	95.89	–14.83

TABLE 3. LCN results for p=2; dipole with L = 0.48λ, a = 0.0391λ, b/a = 1.187					
cells on barrel	cells on each end	unknowns	equations	R_{in}	X_{in}
26	1	56	56	99.74	–4.90
50	2	108	108	98.45	–9.25
74	3	160	160	97.96	–10.92
98	4	212	212	97.71	–11.79
124	5	268	268	97.57	–12.35
148	6	320	320	97.49	–12.69
172	7	372	372	97.41	–12.95
196	8	424	424	97.34	–13.15
222	9	480	480	97.26	–13.33
246	10	532	532	97.18	–13.47

TABLE 4. LCN results for p=6; dipole with L = 0.48λ, a = 0.0391λ, b/a = 1.187					
cells on barrel	cells on each end	unknowns	equations	R_{in}	X_{in}
12	1	84	84	97.88	–10.96
24	2	168	168	97.40	–12.74
36	3	252	252	97.22	–13.35
48	4	336	336	97.12	–13.67
60	5	420	420	97.06	–13.87
72	6	504	504	97.02	–14.00

TABLE 5. LCN results for p=9; dipole with L = 0.48λ, a = 0.0391λ, b/a = 1.187					
cells on barrel	cells on each end	unknowns	equations	R_{in}	X_{in}
12	1	126	126	97.37	–12.84
24	2	252	252	97.11	–13.72
36	3	378	378	97.02	–14.02
50	4	522	522	96.98	–14.19