Backward Differentiation (BDF) - based Numerical Schemes for Efficient Time Integration of Maxwell's Equations

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The Finite Difference Time Domain (FDTD) technique presents a versatile and relatively simple way of solving Maxwell's equations in the time-domain, for arbitrary geometries. Yet, two fundamental concerns limit its applicability to multi-wavelength domain problems, of the type that often arise in wireless communication applications. First, numerical dispersion necessitates the use of dense spatial gridding. Second, numerical instability necessitates the use of a dense temporal gridding. An answer to the first concern, comes from the so-called higher order methods, such as S-MRTD [Krumpholz and Katehi, IEEE MTT-T, April 1996]. An important matter that little attention has been given to, is that their combination with low-order time integration schemes (such as the FDTD leap-frog, or midpoint rule), compromises their accuracy [Aidam and Russer, Proc. 15th Annu. Rev. Progress App. Comput. Electromag.].

In this paper, we focus on the initial value problem that comes from the space discretization of Maxwell's equations with higher order functions, namely Battle-Lemarie, biorthogonal and Daubechies scaling functions. Recognizing that what is needed for efficient time integration of the resultant IVP is a stable, higher order method, we adopt an effective class of methods that has been previously employed in the so-called *stiff* problems of fluid dynamics: Backward differentiation methods [Curtis and Hirschfelder, Proc. Nat. Acad. Sci. USA, 1952]. The latter, approximate temporal derivatives at time step $n+r$ by a backward difference approximation, using r additional points going backwards in time. It is noted that the derivation of an $r-th$ order accurate method, using r points is possible. The 1-step BDF is simply the well-known first order accurate, backward Euler method. The highest order of accuracy that can be implemented for a stable BDF is six.

Dispersion analysis of all schemes under consideration is provided and stability conditions are explicitly stated. Derivations of absorbing boundary conditions are presented. Benchmarking applications to cavity problems, as well as real-life applications such as integrated waveguides and antennas are considered.