Accurate Metal Edge Processing for Edges Parallel to the FDTD Grid

C. Whelan*, M. Okoniewski, University of Calgary, Canada

Modelling geometries within the Finite Difference Time Domain (FDTD) technique is challenging. Cartesian mesh allows for easy formulation of update equations, yet can cause difficulty when the problem geometry does not coincide with the grid. One particular case is that of a sharp metal edge. Singularities in field exist near the edge and if taken into account, they can increase the accuracy of an FDTD simulation involving flat metals. Currently, static cases, such as metals at a 45° angle to the mesh (Esselle, Okoniewski, and Stuchly *IEEE Microwave and Guided Wave Letters*, Vol. 9, No. 6, June 1999) have been solved. These techniques have been extended using a graded mesh to model metal edges lying at different angles. The drawback with these techniques is the moving of H field components from their cell-centered position, requiring a re-formulation of E and H updates for that cell. While solving the problem of modeling arbitrarily angled flat metals using only altered updates for H fields, equations have been developed which take into account the singularity near the metal edge and can be applied when it runs *parallel but not aligned with* the cartesian FDTD grid. These equations are defined on the transverse electric plane for two cases. The first case occurs when the H field is located on or inside the metal strip; the update equation is:

$$H_z^{i,j,k} = 0 \tag{1}$$

The second case occurs when the metal edge is between the cell edge (E component), and the cell center(H component) with the H component located outside of the metal. The update equation is:

$$H_z^{i,j,k} = H_z^{i,j,k} - \frac{\Delta t}{\mu}\left(\frac{E_x^{i,j+1,k} - E_x^{i,j,k}}{\Delta y(2\sqrt{l^2 - \frac{1}{2}l})} - \frac{E_y^{i+1,j,k} - E_y^{i,j,k}}{\Delta x}\right) \tag{2}$$

where l is the percentage of the cell's perpendicular side that is located outside of the metal. If the metal sheet edge falls directly on the E field component (i.e. $l = 0.0$) then the $2\sqrt{l^2 - \frac{1}{2}l}$ in the denominator becomes $\sqrt{2}$. As the edge approaches the H component in the center of the cell, l approaches 0.5, and the denominator of the corresponding spatial derivative approaches zero, reflecting the singular behaviour of the field in the vicinity of the edge. These update equations have been implemented and applied to a simple microstrip circuit. Without modified update equations, the impedance of the strip is calculated to be approximately 39.54 ohms at 10GHz, 41.4 ohms at 20 GHz, and 41.66 ohms at 30 GHz. With the above equations, the microstrip was shifted across the width of one cell and impedance measurements were taken for each shift. The modified equations did not cause instability, and the results in Table 1 show the modified equations' efficacy.

Table 1: Impedance results for modified equations at three frequencies.

l	Z_{10GHz}	Z_{20GHz}	Z_{30GHz}
0.3	42.45	41.80	40.30
0.6	39.58	41.34	41.68
0.9	39.58	41.36	41.72