Leaky Surface-Plasmon Theory for Dramatically Enhanced Transmission through a Sub-Wavelength Aperture,
Part II: Leaky-Wave Antenna Model

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I. Introduction

The optical transmission through an aperture punched in an opaque metal (usually silver) film is extremely small when the aperture diameter d is much smaller than the wavelength, $d \ll \lambda_0$. The optical transmission can be enhanced greatly when one or both surfaces of the metal film have periodic corrugations, provided the corrugations are chosen properly, as described in various papers, for example [1, 2], and discussed in the companion paper, Part I. The general structure is shown in Fig. 1.

The best explanation to date for the dramatically enhanced transmission effect has been in terms of a resonance associated with a surface plasmon mode that exists on the metal film. A surface plasmon is essentially a surface-wave type of mode that propagates on the surface of a metal at optical frequencies, where the relative ***permittivity*** of the metal is ***negative.*** (See the discussion in Part I.) According to the best existing explanation for the effect, a resonance occurs that produces a standing wave when the periodicity is chosen according to the condition

$$\beta^{P} a = 2\pi \qquad (1)$$

where a is the period, and β^{P} is the phase constant of the surface plasmon mode on a smooth surface. As pointed out in the companion paper, Part I, a more accurate condition that resembles (1) actually corresponds to a leaky mode that radiates at broadside, and does not correspond to a resonance or a standing wave. (See Eq. (3) of Part I.)

The purpose of this paper is to demonstrate, for the first time, that the enhanced optical transmission effect is indeed due to the excitation of a leaky mode. In particular, the periodic structure (corrugations) causes the normally bound (non-leaky) surface plasmon mode to become a leaky plasmon mode. By properly choosing the periodicity, the leaky mode radiates at broadside, giving an increased field in this direction, and hence an enhanced optical transmission.

This ***new point of view*** allows for new ways to optimize and design the structure, based on the phase and attenuation constants of the leaky mode, β^{LM} and α^{LM}, respectively. Because of the loading effect of the periodic structure, β^{LM} is not the same as the phase constant β^{P} of the surface plasmon mode that exists without the periodic structure, so that β^{P} in (1) should be replaced by β^{LM} for a radiation peak at broadside. The leaky-mode theory also allows for an easy estimation of how large a practical finite structure needs to be in order to realize the dramatically enhanced transmission effect, based on α^{LM}, as discussed in Part I.

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In the present study, the periodic corrugations have been replaced by a periodic array of perfectly conducting metal patches on the surface of the silver film, for ease of analysis. The physics of the transmission enhancement due to the periodic structure should remain the same, however.

II. Analysis

Consider the structure shown in Fig. 2, which is essentially a leaky-wave antenna excited by a horizontal y-directed magnetic dipole (which models the exit aperture in Fig. 1, assuming that the electric field in the aperture is polarized in the x direction). A factor R is defined as the ratio of the power radiated at broadside by the dipole with the periodic structure on the exit face only, to the power radiated at broadside without the periodic structure. The factor R is larger than unity due to the radiation from the leaky mode.

By reciprocity, it can be shown that on the entrance aperture, the square of the electric field strength is enhanced by this same factor R. Hence, the overall enhancement in the power transmission is R^2. The factor R can be determined by calculating the far-field radiation pattern of the leaky-wave antenna shown in Fig. 2, using a periodic moment-method solution together with reciprocity [3]. The details are omitted here. A periodic moment-method solution is also used to find the complex wavenumber, $\beta^{LM} - j\,\alpha^{LM}$, of the leaky plasmon mode.

III. Results

Results are shown for a lossless silver film with h = 300 nm, and a relative permittivity of $\varepsilon_r = -4.5$ at a wavelength of 399.72 nm (the frequency is 7.5 E+14 Hz). An actual silver film would have mild loss at optical frequencies, but a lossless film is assumed in these calculations. The metal patches have a length L = 140 nm in the x direction and a width W = 50 nm in the y direction. The period in the y direction is b = 90 nm. The period a in the x direction is specified in the figures.

Figure 3 shows (in polar form) the E-plane pattern (in the xz plane) for the structure in Fig. 2, where an optimum period a = 377 nm was found from a numerical optimization, where the period was changed until a maximum power radiated at broadside was found. Also included in Fig. 3 is the pattern of the same magnetic dipole on the silver layer without the periodic patch array. Both patterns are normalized so that the maximum of the pattern in the periodic case is zero dB. Clearly, the presence of the periodic structure has resulted in a very narrow-beam radiation pattern with significantly enhanced radiation (about 22 dB) at the peak of the beam, due to the leaky mode.

Figure 4 shows a comparison of normalized E-plane radiation patterns (on a rectangular scale) near the beam maximum for the case of the optimum period in Fig. 3. In this figure the exact total pattern is compared with the pattern of the leaky mode alone. Both patterns are normalized to zero dB at the maximum. The leaky-mode pattern is obtained from the complex wavenumber of the leaky mode, using the wavenumber to approximate the current amplitude on the metal patches. A simple closed-form array factor expression is then obtained from these currents. It is seen that there is good agreement between the exact and leaky-mode patterns, verifying that the narrow-beam pattern is indeed due to a leaky mode.

Figure 5 shows a comparison of the numerically optimized pattern with the pattern obtained when the period a = 353 nm is used, as found from Eq. (1) with β^P. It is seen that using β^P in Eq. (1), which neglects the loading effect of the periodic structure on the wavenumber of the leaky mode, gives a pattern with peaks that are scanned away from broadside. Clearly, this is not an optimum pattern. (See the corresponding discussion in Part I.)

IV. Conclusions

It has been shown that the dramatically enhanced optical transmission effect for small apertures in a silver film at optical frequencies surrounded by a periodic structure is due to a leaky mode. This new viewpoint allows for an improved design of the periodic structure, and provides much physical insight into the phenomenon.

References

[1] T. Thio, H. J.Lezec, and T. W. Ebbesen, "Strongly Enhanced Optical Transmission Through Subwavelength Holes in Metal Films," *Physica B*, 279, pp. 90-93, 2000.

[2] H. J. Lezec, A. Degiron, E. Devaux, R. A. Linke, L Martin-Moreno, F. J. Garcia-Vidal, and T. W. Ebbesen, "Beamimg Light from a Subwavelength Aperture," *Science,* Vol. 297, pp. 820-822, 2 August 2002.

[3] T. Zhao, D. R. Jackson, J. T. Williams, and H. Y. Yang, "Radiation Characteristics of a 2D Periodic Leaky-Wave Antenna using Metal Patches or Slots," *IEEE AP-S Intl. Symp*., Boston, MA, pp. 260-263, July 8-13, 2001.

Fig. 1. Geometry of metal film layer (silver) at optical wavelengths with corrugations on both faces, and apertures connected by a small hole in the film.

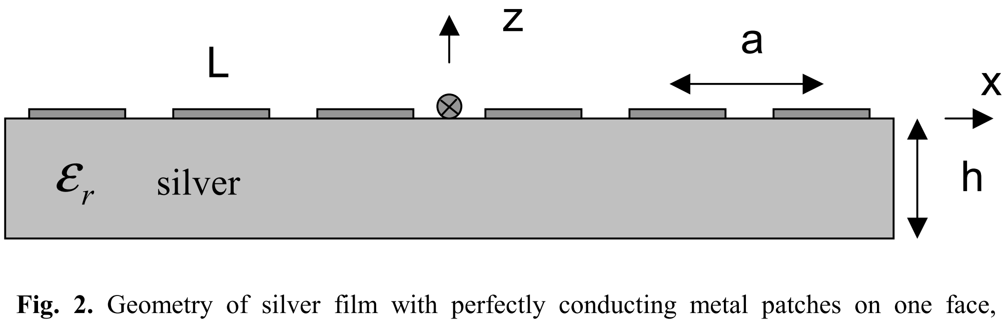

Fig. 2. Geometry of silver film with perfectly conducting metal patches on one face, excited by a horizontal magnetic dipole in the y direction at the location of the original aperture. The patches have length L and width (not shown) W.

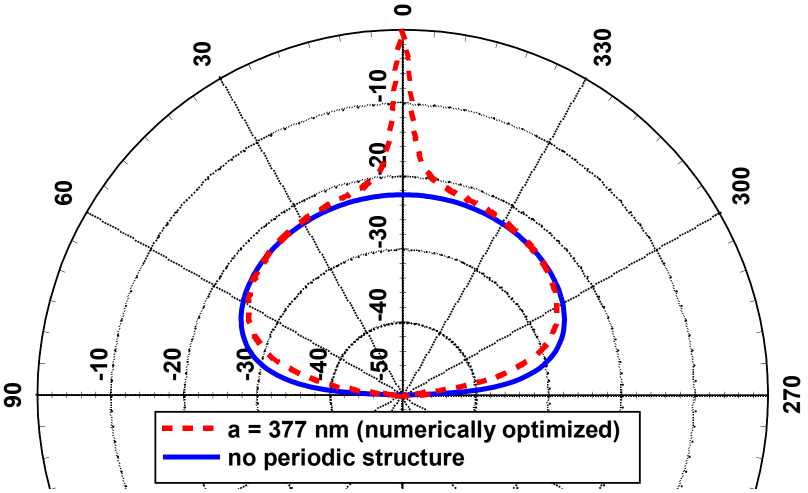

Fig. 3. Polar plot of the E-plane pattern for the structure in Fig. 2. The period *a* = 377 nm is the numerically optimized value. Note that without the periodic structure, the beam is very broad, and that with it, the peak at broadside is increased by about 22 dB.

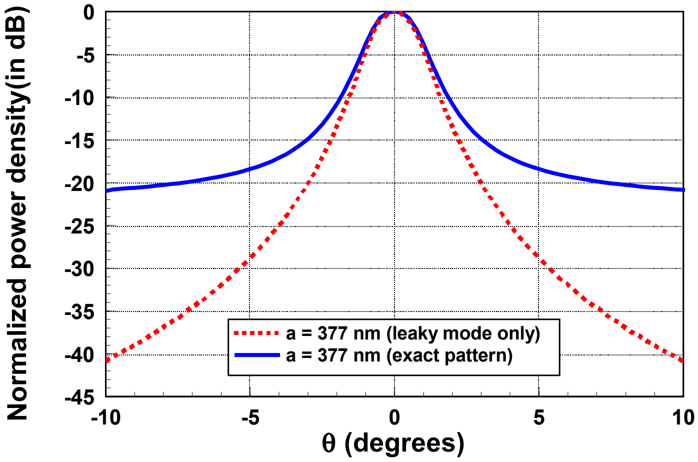

Fig. 4. Radiation patterns near broadside, for the structure in Fig. 2. The exact pattern is compared with the leaky-mode pattern for *a* = 377 nm. Both patterns are normalized to zero dB at the maximum. The peak at broadside is clearly due to the leaky mode.

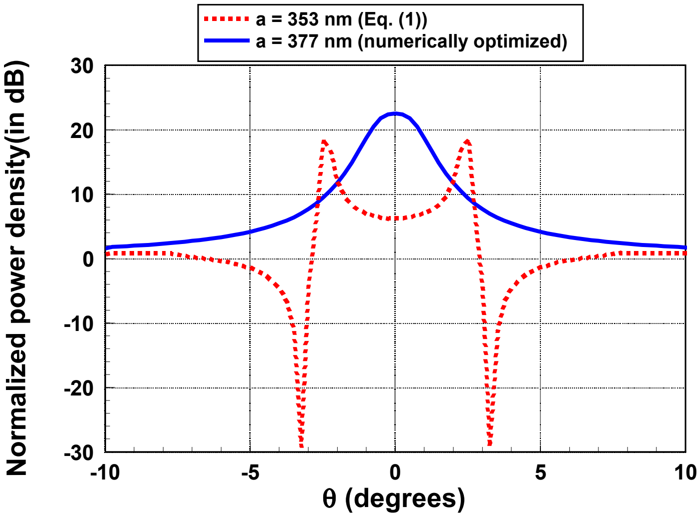

Fig. 5. Radiation patterns near broadside, for the structure in Fig. 2. The power density at each angle has been normalized to that radiated at broadside by the same structure without the periodic patch structure, and then expressed in dB. The period *a* = 353 nm corresponds to Eq. (1) with β^P (red dotted curve), which does not produce a peak at broadside. See text for fuller explanation.