A Fast Hybrid MoM/FEM Technique for Microstripline Vertical Couplers With multiple Identical Cavities

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1 Introduction

A hybrid technique [1] has successfully modeled cavity-backed patch antennas [1] and microstripline-fed slot-coupled resonant cavities[2]. This technique primarily follows the MoM, except in the cavity the Green's function is replaced by the solutions of FEM such that the exact Green's function is not needed. Due to the flexibility of FEM, the shape of the cavity is not restricted to be rectangular or cylindrical and the cavity can be filled with complex material. Thus, this technique preserves the efficiency of MoM while retains the flexibility of FEM.

Base on the above technique, the formulation of multiple cavities is derived in this paper. A novel technique to reduce the computational cost to the case of only one cavity when all cavities are identical is introduced. Comparison of simulation results and experimental data are also shown.

2 Formulation

A single cavity coupler is shown in Fig. 1. Two slots are open on the top and bottom of the metallic cavity that is sandwiched between two layers of dielectric substrates. Either one of the microstrip lines passing the slots can be used as the input line or output line of this structure. Without loss of generality, we choose the lower microstripline as the input line.

Consider the structure in Fig. 2. Instead of single cavity, n cavities are stacked together and coupled through slots on their common walls. One way of analyzing this structure is considering the n-cavity structure as one big cavity and using similar formulation as in [1]. However, it is more efficient to perform FEM computation separately on each cavity instead of on the whole cavity.

By closing all slots and applying the equivalence principle as shown in Fig. 2, we can derive the following matrix equations:

$$\begin{aligned}
\begin{bmatrix} V_{inc} \\ 0 \end{bmatrix} + \begin{bmatrix} Z_f & 0 \\ 0 & Z_t \end{bmatrix} \begin{bmatrix} J_f \\ J_t \end{bmatrix} &= \begin{bmatrix} T_f & 0 \\ 0 & T_t \end{bmatrix} \begin{bmatrix} M_1 \\ -M_{n+1} \end{bmatrix}, && (1) \\
[C_{inc}] + [C_f]\,[J_f] &= [Y_{11} + Y_f]\,[M_1] - [Y_{12}]\,[M_2], && (2) \\
[Y_{21}]\,[M_1] - [Y_{22}]\,[M_2] &= [Y'_{22}]\,[M_2] - [Y_{23}]\,[M_3], && (3) \\
\vdots \quad & \vdots \quad \vdots \\
[Y_{i,i-1}]\,[M_{i-1}] - [Y_{i,i}]\,[M_i] &= \left[Y'_{i,i}\right][M_i] - [Y_{i,i+1}]\,[M_{i+1}], && (4) \\
\vdots \quad & \vdots \quad \vdots \\
[Y_{n,n-1}]\,[M_{n-1}] - [Y_{n,n}]\,[M_n] &= \left[Y'_{n,n}\right][M_n] - [Y_{n,n+1}]\,[M_{n+1}], && (5) \\
[Y_{n+1,n}]\,[M_n] - [Y_{n+1,n+1} + Y_t]\,[M_{n+1}] &= [C_t]\,[J_t], && (6)
\end{aligned}$$

where J_f, J_t, and $M_i (i = 1, 2, ..., n + 1)$ are the expansion coefficients of the unknown currents $\bar{J}_f$, $\bar{J}_t$, and $\bar{M}_i$ as shown in Fig. 2. The meaning of the matrices is tabulated in Table 1.

Note that, we use the same basis functions on the microstriplines as described in [3]. This allows us to derive the reflection and transmission coefficients of the input and output lines directly.

Note that $[Y_{i,i}]$ and $\left[Y'_{i,i}\right]$ are the self interactions of $\bar{M}_i$ in its lower and upper cavity respectively.

Outside the Cavities

Matrix	Interaction between	
V_{inc}	J_f	J_{inc}
C_{inc}	$\bar{M}_1$	$\bar{J}_{inc}$
Z_f	$\bar{J}_f$	$\bar{J}_f$
Z_t	$\bar{J}_t$	$\bar{J}_t$
T_f	$\bar{J}_f$	$\bar{M}_1$
T_t	$\bar{J}_t$	$\bar{M}_{n+1}$
C_f	$\bar{M}_1$	$\bar{J}_f$
C_t	$\bar{M}_{n+1}$	$\bar{J}_t$
Y_f	$\bar{M}_1$	$\bar{M}_1$
Y_t	$\bar{M}_{n+1}$	$\bar{M}_{n+1}$

Inside the Cavities

Matrix	Interaction between	
Y_{ij}	M_i	M_j
Y_{ii}	$\bar{M}_i$	$\bar{M}_i$
Y'_{ii}	$\bar{M}_i$	$\bar{M}_i$

Table 1: The meanings of matrices in(1) to (6)

Unlike the formulation of single-cavity case, the above equation is very cumbersome to solve by manipulating the matrices. Instead, by reassembling (1) to (6), we can solve this equations in a single formula at the expense of large matrix size.

If the cavities and the slots are all the same, all the unknowns can be solved by using the interactions of the single cavity structure.

Assuming the cavities and the slots are all the same, we have

$$[Y_{i,i-1}] = [Y_{i,i+1}] = A, \ \ [Y_{i,i}] = \left[Y'_{i,i}\right] = B, \tag{7}$$

for $i = 2, \ldots, n$. To simplify these equations, we use M_i instead of $[M_i]$. (4) becomes

$$AM_{i-1} - BM_i = BM_i - A_{i+1}, \ \ \Rightarrow \quad M_{i-1} - B'M_i = B'M_i - M_{i+1}, \tag{8}$$

where $B' = A^{-1}B$. Assume (8) can be recast into the following recursive form:

$$M_{i-1} - bM_i = c(M_i - bM_{i+1}). \tag{9}$$

We find c and b satisfy

$$b + c = 2B' \quad \text{and} \quad cb = 1. \tag{10}$$

Substitute the recursive relationship to (3)–(5), we can derive the following formula:

$$M_1 - bM_2 = c^{n-1}(M_n - bM_{n-1}), \tag{11}$$

$$M_1 - cM_2 = b^{n-1}(M_n - cM_{n-1}). \tag{12}$$

Solving (11) and (12) simultaneously, we can derive M_2 and M_n in terms of M_1 and M_{n+1} as follows:

$$\begin{array}{rcccl} M_2 & = & \frac{(b^{n-1}-c^{n-1})M_1+(b-c)M_{n+1}}{b^n-c^n} & = & \alpha M_1 + \beta M_{n+1}, \\ M_n & = & \frac{(b-c)M_1+(b^{n-1}-c^{n-1})M_{n+1}}{b^n-c^n} & = & \beta M_1 + \alpha M_{n+1}. \end{array} \tag{13}$$

Substituting (13) to (2) and (6), we have

$$\left[\begin{array}{c} C_{inc} \\ 0 \end{array}\right] + \left[\begin{array}{cc} C_f & 0 \\ 0 & C_t \end{array}\right] \left[\begin{array}{c} J_f \\ J_t \end{array}\right] = \left[\begin{array}{cc} Y_1 - Y_{12}\alpha & -Y_{12}\beta \\ -Y_{n+1,n}\beta & Y_{n+1} - Y_{n+1,n}\alpha \end{array}\right] \left[\begin{array}{c} M_1 \\ M_{n+1} \end{array}\right]. \tag{14}$$

With (14) and (1) the unknowns J_f, J_t, M_1 and M_{n+1} can be solved easily as in the case of single cavity. Other unknowns can be derived from M_1 and M_{n+1} recursively.

Note that in (13) α and β are evaluated in terms of c and b. However, c and b may be difficult to derive because of the square root operation of matrices is involved. Instead, we can derive α and β in terms of $b + c$ and bc.

By taking out the common factor $b - c$ in (13), α and β become

$$\alpha = \frac{S_{n-2}}{S_{n-1}},\ \beta = \frac{1}{S_{n-1}}, \tag{15}$$

where

$$S_n = (b + c)S_{n-1} - bcS_{n-2} \quad \text{for } n \geq 1 \tag{16}$$

if we let $S_{-1} = 0$ and $S_0 = 1$.

With (15), (16) and (10), α and β can be derived solely from the A and B in (7) without solving b and c directly. To check the validity of these equations, set $n = 1$. We find $\alpha = 0$ and $\beta = 1$. After Substituting this result into (14), this equation becomes the same as single cavity formulation.

3 Validation

In this section, the computed result using the single-cavity formulation is compared to the computed and experimental data of [4]. In that paper, a microstripline coupler with thick ground plane was investigated. The cavity was treated as a rectangular waveguide and only the TE_{10} mode was used in its MoM formulation. Outside the cavity, quasi-static Green's functions were used in its MoM formulation.

The first and second £gures in Fig. 3 show the comparison between the paper's result and the computed result of the hybrid technique for two different ground plane thicknesses. Good agreement with measurement is achieved for both thicknesses. In the 50 mil case, the result of the hybrid technique is closer to the experimental data than the computed data of [4]. This is because only TE_{10} mode modeling is not enough when the ground plane is thin.

Since 1000 mil is multiples of 50 mil, we can use the multi-cavity formulation to derive the computed result of 1000 mil case from 50 mil case. Fig. 3 shows the result of considering the 1000 mil as twenty 50 mil cavities stacked together. The computed $|S_{12}|$ is almost the same as the single cavity model except for a small deviation of $|S_{11}|$ near resonance.

4 Conclusion

In this chapter, the hybrid MoM/FEM method is successfully applied to a microstripline-fed slot-coupled cavity. The computed results are in good agreement with the measurement. A novel technique to solve the MoM matrices of multiple identical cavities from the result of single cavity is successfully demonstrated.

References

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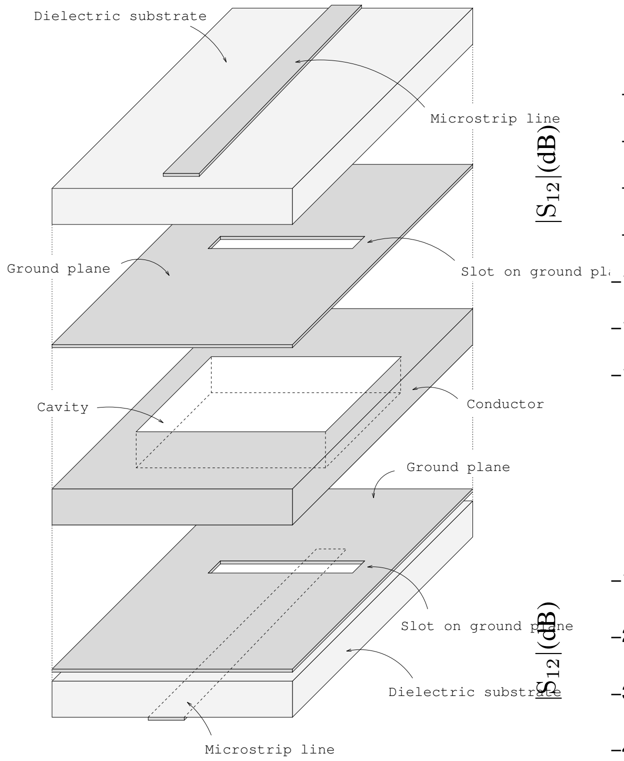

Figure 1: The structure of a microstrip cavity coupler.

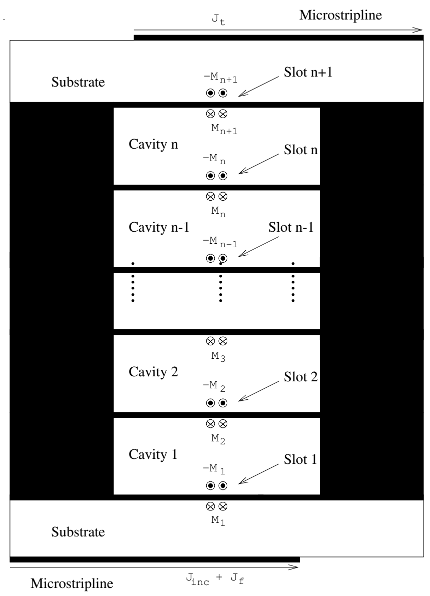

Figure 2: The side view of a multi-cavity coupler.Note that the slots on the cavity are closed by a PEC with an equivalent magnetic current.

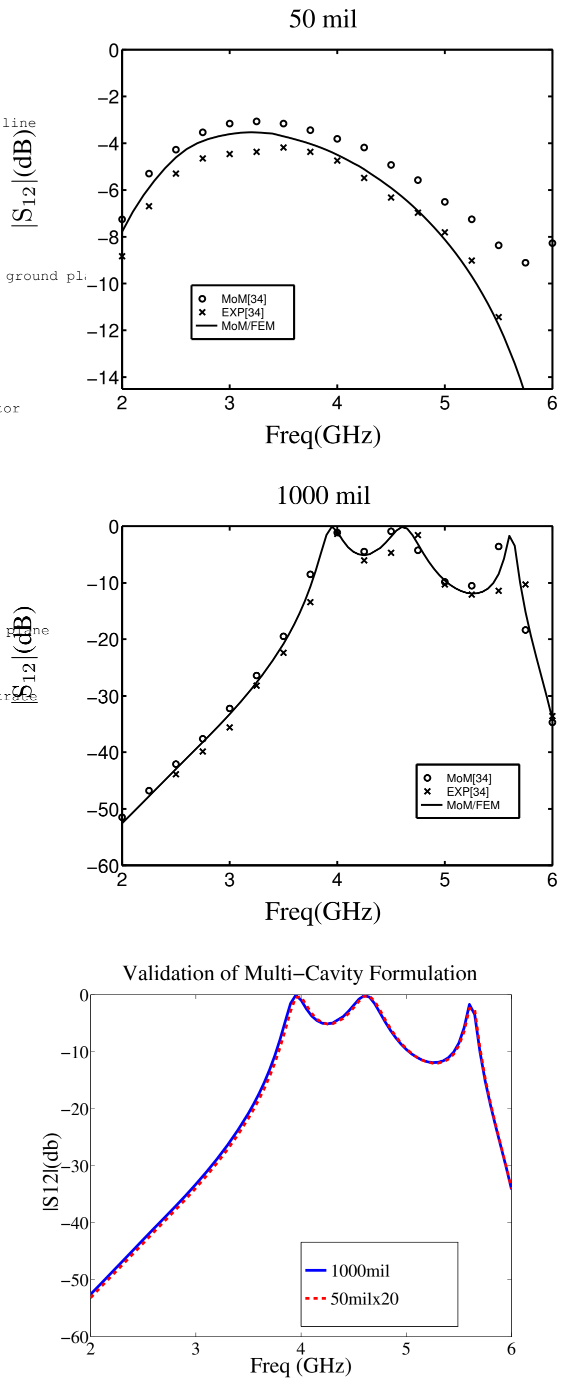

Figure 3: The first and second £gures are the comparison of the result from comparison of the computed result of the single cavity and multi-cavity formulations. The solid line is the data computed by the single-cavity model and the dashed line is the data computed from the multi-cavity model.