Nédélec's Element Definition on Simplex Coordinates

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Among the curl-conforming elements appeared in the Finite Element Method (FEM) literature, it is worth mentioning the so called mixed order elements proposed by J. C. Nédélec (*Numerische Mathematik*, 35, 315–341, 1980). The authors have proposed a methodology for the construction of the k-th order finite element vector basis functions over 2D and 3D simplices that follows rigorously the finite element definition given by Nédélec. The finite element is defined in terms of a domain, a space of functions $\mathcal{R}^k$ and a set of degrees of freedom acting as linear functionals on $\mathcal{R}^k$. The vector basis functions are obtained by imposing the interpolatory character of the basis functions with respect to the definition of the degrees of freedom of the element, which act as linear functionals on $\mathcal{R}^k$. The application of this methodology has leaded to a mixed-order curl-conforming family of finite elements (see M. Salazar-Palma et al., *Iterative and Self-Adaptive Finite-Elements in Electromagnetic Modeling*, Artech House, 1988, L. E. García-Castillo et al., *Int. J. Num. Modelling: Elec. Networks, Devices and Fields*, 13, 261–287, 2000, L. E. García-Castillo et al., *IEEE Trans. Magnetics*, 38. 2370–2372, 2002). Higher-order triangular and tetrahedral elements up to third-order, have been implemented using this methodology with cartesian coordinates. Vector basis functions are obtained in the master element and, by means of an affine transformation, are mapped to the real element.

The objective of this paper is to explore the application of the author's methodology with simplex coordinates (also called natural coordinates), i.e., the definition of $\mathcal{R}^k$ on simplex coordinates and the imposition of the definition of the degrees of freedom using simplex coordinates. A comparison between both approaches, i.e., finite element definition using cartesian and simplex coordinates, will be discussed, and the advantages and disadvantages of both approaches will be shown.