Characteristic Basis Function Method for Solving Large Problems Arising in Dense Medium Scattering

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Introduction

Electromagnetic scattering from a dense medium consists of a large number of dielectric scatterers is of great interest. In natural media such as snow and ice, densely packed and electrically small dielectric scatterers are randomly distributed in a host medium [1]. On the other hand, these scatterers can be arranged regularly, in a lattice form, in man-made materials such as the photonic bandgap structure [2]. For these dense media, multiple scattering and coherent wave mutual interactions must be taken into account and many exiting analytical and approximation theories may require the use of pair distribution function and/or configurational symmetries. An alternative approach is to resort to more rigorous numerical methods such as the Method of Moments (MoM) at the expense of extensive CPU time and memory [3].

Recently, several approaches have been proposed for matrix size reduction [4]-[5]. These approaches entail the analysis of partial domains of the original problem for the construction of Macro Basis Functions (MBFs). These MBFs, however, do not take into account of mutual couplings among all the partial domains. To incorporate mutual coupling effects, the use of Characteristic Basis Functions (CBFs) has recently been proposed [6], [7]. In this method, the mutual coupling effects are included through the use of higher-level basis functions, referred to the primary and secondary CBFs. The coefficients of these CBFs are solved for directly using the Galerkin method. In this paper, we implement the CBF method for dense medium scattering. These CBFs, however, are constructed differently using the Foldy-Lax equations [3] in which mutual coupling effects among all scatterers can be included systematically. Our results in this paper show that a small number of CBFs is sufficient.

Construction of Characteristic Basis Functions

Assuming that we have N dielectric scatterers that are distributed in a cubic volume and each scatterer is subdivided into m dielectric cells. The tensor integral

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equation for the electric field inside the dielectric body can be transformed into a matrix equation using MoM as

$$\begin{bmatrix} \overline{\overline{Z}}_{11} & \overline{\overline{Z}}_{12} & \overline{\overline{Z}}_{13} & \cdots \overline{\overline{Z}}_{1N} \\ \overline{\overline{Z}}_{21} & \overline{\overline{Z}}_{22} & \overline{\overline{Z}}_{23} & \cdots \overline{\overline{Z}}_{2N} \\ \overline{\overline{Z}}_{31} & \overline{\overline{Z}}_{32} & \overline{\overline{Z}}_{33} & \cdots \overline{\overline{Z}}_{3N} \\ \vdots & \vdots & \vdots & \cdots \vdots \\ \overline{\overline{Z}}_{N1} & \overline{\overline{Z}}_{N2} & \overline{\overline{Z}}_{N3} & \cdots \overline{\overline{Z}}_{NN} \end{bmatrix} \begin{bmatrix} \vec{J}_1 \\ \vec{J}_2 \\ \vec{J}_3 \\ \vdots \\ \vec{J}_N \end{bmatrix} = \begin{bmatrix} \vec{E}_1^{inc} \\ \vec{E}_2^{inc} \\ \vec{E}_3^{inc} \\ \vdots \\ \vec{E}_N^{inc} \end{bmatrix} \tag{1}$$

where $\vec{J}_i$ and $\vec{E}_i^{inc}(i = 1,2,\cdots N)$ are the induced volumetric current densiity and incident field sampled at the center of each cell, respectively, and $\overline{\overline{Z}}_{ij}(i = 1,2,\cdots N; j = 1,2,\cdots N)$ are *3m by 3m* sub-matrices. We can reduce this matrix size substantially through the use of characteristic basis functions obtained by the Foldy-Lax multiple scattering equations [3]. The equations state that the final exciting field of the *i-th* scatterer is equal to the incident field plus scattered fields from all scatterers except the scattered field from itself. Using the Foldy-Lax equations, the primary CBF for each scatterer corresponds to the incident field only, and its construction ignores the scattered fields due to all other scatterers, as though the scatterer was isolated. On the other hand, the first secondary CBF on a scatterer is computed by replacing the incident field with the scattered fields due to primary CBFs on all scatterers except from itself. Similarly, additional secondary CBFs can be computed. These CBFs are written as follows:

$$\overline{\overline{Z}}_{ii}\vec{J}_i^P = \vec{E}_i^{inc} \quad (i = 1,2,\cdots N) \tag{2}$$

$$\overline{\overline{Z}}_{ii}\vec{J}_i^{S1} = -\sum_{j=1(j\neq i)}^{N} \overline{\overline{Z}}_{ij}\vec{J}_j^P \quad (i = 1,2,\cdots N) \tag{3}$$

$$\overline{\overline{Z}}_{ii}\vec{J}_i^{S2} = -\sum_{j=1(j\neq i)}^{N} \overline{\overline{Z}}_{ij}\vec{J}_j^{S1} \quad (i = 1,2,\cdots N) \tag{4}$$

$$\vec{J}_i^t = a_i\vec{J}_i^P + b_i\vec{J}_i^{S1} + c_i\vec{J}_i^{S2} \quad (i = 1,2,\cdots N) \tag{5}$$

The primary CBFs can be derived easily using conventional matrix inversion and the secondary CBFs can be computed by backsubstitution after Z_{ii} are factorized. The matrix size in Eq. (1) is now substantially reduced when $\vec{J}_i$ is replaced by $\vec{J}_i^t$ as the number of unknowns in each scatterer is now the number of CBFs employed in Eq. (5). The new matrix elements are in the form of $(\vec{J}_i^{P,S})^T \bullet (\overline{\overline{Z}}_{ij}\vec{J}_j^{P,S})$ with superscript T stands for the transpose operation. The same set of equations applies to the situation when a cluster of scatterers replaces each scatterer.

Numerical Results

To illustrate the efficiency and accuracy of the proposed method, we consider the scattering problem of 125 dielectric scatterers of $0.2\lambda_o$ x $0.2\lambda_o$ x $0.2\lambda_o$ in size with

a dielectric constant of 4 randomly distributed in a volume of $2.154\lambda_o$ x $2.154\lambda_o$ x $2.154\lambda_o$, making the fractional volume 10%. Each scatterer is subdivided into 64 small cubes, yielding a total of 24,000 unknowns. We compare our results against the sparse-matrix canonical grid method and both algorithms are implemented on a 16-PC cluster using Pentium III 667 MHz processors. Figure 1 shows the good agreement in the x component of the current distributions computed by SMCG and the proposed method. Figures 2 and 3 shows the convergence of the scattered fields E_ϕ and E_θ versus the number of CBFs, respectively. Excellent agreement with results obtained from SMCG is demonstrated. Comparison of the CPU time versus the number of CBFs is given in Table I. The CPU time for the SMCG is 1151 seconds. For the SMCG method, we solve a matrix equation of 24,000. In contrast, the proposed method requires solving a matrix equation of 64x3 = 192 unknowns 125 times for each set of CBFs and a final matrix equation of 125x4=500 unknowns for the weighting coefficients of the 4 CBFs.

Acknowledgement

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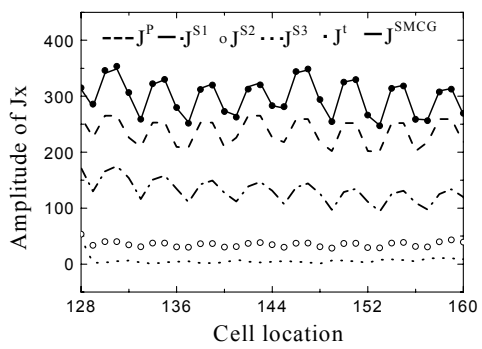


Figure 1. Comparison of $|J_x|$ in selected locations.

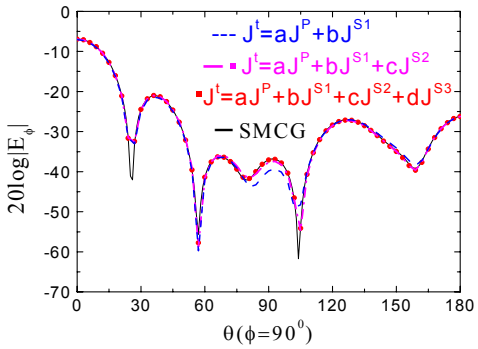
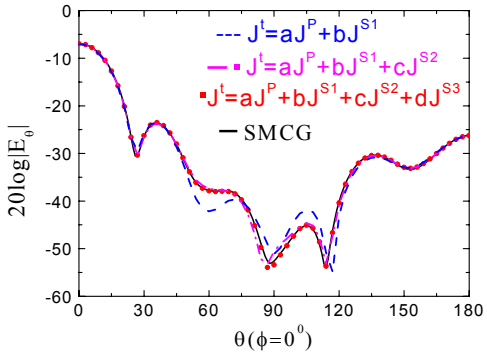
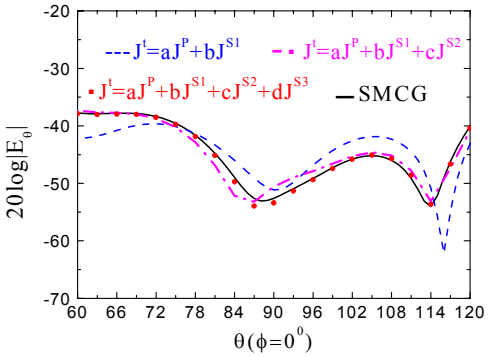


Figure 2. Convergence of E_ϕ vs the numberof CBFs.



(a)



(b)

Figure 3. Convergence of E_θ vs the number of CBFs. (a) Original curves and (b) enlarged curves.

TABLE I. COMPARISON OF CPU TIME AND THE NORMALIZED L-2 NORM OF THE ORIGINAL MATIX EQUATION IN EQ. (1).

	CPU time	Normalized L-2 norm
$J^t = aJ^P + bJ^{S1}$	128.11s	22.46%
$J^t = aJ^P + bJ^{S1} + cJ^{S2}$	171.29s	8.23%
$J^t = aJ^P + bJ^{S1} + cJ^{S2} + dJ^{S3}$	213.25s	2.59%