MATRIX COMPRESSION AND SUPERCOMPRESSION TECHNIQUES FOR LARGE ARRAYS

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1. INTRODUCTION

This paper deals with design-oriented analysis based on matrix compression techniques for large array antennas. We will consider arrays with planar radiating elements metalized on dielectric stratification ("printed" arrays of patches, slots, etc.), or made of non-planar conductors in air (with or without backplane), as well as waveguide-based aperture arrays. Reflectarrays, though not explicitly mentioned, are typical applications for both classes.

Array modeling issues are challenging, since they involve large (in terms of the wavelength) structures, but also fine details that require much-smaller than wavelength discretizations, and that dominate the frequency response of input parameters. The Integral Equation (IE) approach is largely used to attack these problems, through the Method of Moment (IE-MoM) discretization scheme. It is well known, however, that standard techniques are severely limited by the matrix size and condition number involved in the problems of interest. In these problems, the structure of the solution exhibits very different scales of variation; for examples, local interactions in a geometry, like sub-wavelength details, edges and discontinuities, generate small-scale details of high spatial frequency, while distant interactions as well as resonant lengths are responsible for the low-frequency, slow spatial variations. One is typically forced to choose mesh cells of size comparable to the smallest foreseen scale of the solution, that is, with the highest possible spatial resolution, or likewise if waveguide modes are used as expansion functions. Unfortunately, this leads to a large number of unknowns, densely populated MoM matrices with a poor condition number, and renders the direct approach of large problems numerically intractable. A number of techniques have been presented in the past years to overcome the above difficulties, whose review is outside the scopes of this work. It is however interesting to note that for conductor-based (EFIE) problems, typical problems of interest were in the area of RCS prediction; in these applications (e.g. [1]), the solution is less beset with the problem of conditioning that arises with sub-wavelength details, and associated difficulties with iterative solvers. We also note that for large arrays of apertures which exhibits periodicity or quasi-periodicity of geometry and excitation, the infinite-extension approximation has been customarily applied, with or without the refinement of "windowing" to account for its finite extent (e.g. [2]). Among all the techniques we will illustrate in details two ones, recently developed in Italy, which are framed in the category of matrix compression, but at a different level of schematization and typology of tractable arrays.

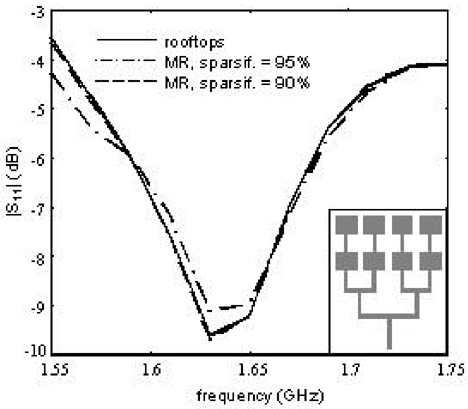

Fig. 1 *Array of patches Frequency response of $|S_{11}|$. Comparison between rooftop basis and DIS-MR scheme+sparsification.*

2. THE SYNTHETIC FUNCTION METHOD

In the Synthetic Function eXpansion (SFX) method [3, and references therein], the structure is broken down into "blocks" (e.g. radiators), and block-global basis functions are obtained on them, that are subsequently used as basis functions in the array analysis. These, called "synthetic functions" (SF), include the small-scale details of the solution, while they are compactly accounted for at inter-element coupling. Few SF are required to correctly represent the current on blocks, and

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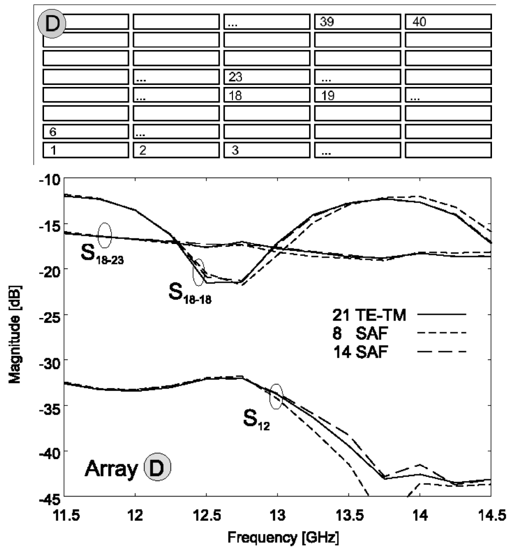

Fig. 2 *Array of 40X40 H-plane stepped horns. Comparison between SAF method and conventional method in calculating scattering parameters (from [5])*

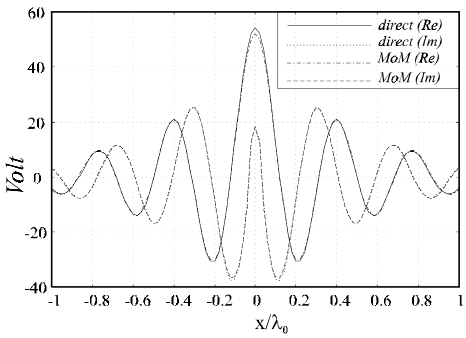

Fig. 2 *Real and imaginary parts of the normalized magnetic current v(x) for an infinite slot (w=0,030) printed on a ground plane between silicon and air, excited by a delta gap with dimension t=0,060. Comparison between closed form Green's function and and fine mesh MoM solution applied to a very long slot (from [5])*

thus the SFX approach reduces the MoM matrix memory occupation, and considerably reduces the time needed to solve the linear system, without affecting the solution accuracy. The SFs are generated numerically from the solution of the stand-alone block structure with appropriate excitation, basing on the equivalence theorem and the limited degrees of freedom of the solution [4]. When computing the interactions of SF on different blocks, the discretization detail can be reduced, leading to a further increase in numerical efficiency.

As an example of application, consider the 4x2 patch array (with BFN) shown in the inset of Fig.1. For this structure, each pair of cascaded patches is a block; deviations below 1% at all frequency points are found in the frequency response of S11 when 6 SFs are employed on each block. In these conditions, memory occupation (at all steps of the process) is about $1/20^{th}$ of that required by a conventional approach, and the linear system solution (with all overhead included) is about $1/10^{th}$ of the standard approach. The SF concept has been applied to arrays of apertures [5]; in this case, waveguide modes are used on apertures, and spatial frequency scales are directly related to modal indices. An example of application for this latter case is shown in Fig. 2. It is concerned with an array of 40X40 H-plane sectoral stepped horns. The synthetic basis function (called here synthetic aperture function, SAF [5]) have been generated by using waveguide mode in isolation excited by the "natural mode" (i.e., the one that feed the element in the actual configuration), plus external excitations to simulate the symmetry breaking due to the array environment. The SAF have been generated in terms of TE and TM mode expansion at the apertures, thus resulting into a compression from a conventional TE-TM MoM matrix of 30-60%. Results for the Sij scattering parameters are presented in Fig. 2 which are obtained with 8 and 14 SAF; results for comparison are obtained by a conventional solution obtained using 21 waveguide modes for every aperture (accepting the results with 8 SAF, the matrix is compressed from 840X840 to 320X320). The improvement of calculation time is evident: to obtain the result in Fig. 2 for a single frequency point by using with 8 SAF, 14 SAF and 21 TE-TM modes, the CPU times was 11", 51", and 2', 51", respectively, being the overhead time for the SAF generation almost negligible.

When applied to an array of printed slot, the synthetic functions can be defined in analytical form by using an exact solution for the transmission line Green's function [6], so to automatically account for the magnetic current deformation around the feeding point. Fig. 3 shows the accuracy of the analitical form of the Green's function on an isolated, infinite slot in comparison with an accurate MoM solution This approach, complemented with resonant SF, is applied in [7] for coplanar waveguides fed slots.

3.TRUNCATED FLOQUET-WAVE MoM

Large and very large arrays are often treated as periodic; strictly speaking, this implies the assumption that the array exhibits a geometrical periodicity and is fed with a linear progressive phase. It is understood that most of the actual arrays are not perfectly periodic neither for the excitation nor for the geometry, but in many practical applications, the deviation with respect to the periodicity conditions is weak – especially when the scale of this deviation is compared to the period of the underlying periodic lattice. Therefore, a large class of three-dimensional (3-D) *finite* phased arrays can be treated as periodic. In these cases, the *infinite* array approximation is often used to reduce the numerical complexity, and is known to yield good results except near the array edges and close to the scan-blindness condition. As an alternative to perturbation approaches based on the above, one can formulate the problem to find explicitly the difference between the solution of the infinite array, and the exact solution. Writing the appropriate (exact) integral equation for this difference (which is termed "fringe" integral equation following a terminology of high-frequency diffraction theories) is one of the starting points of the efficient technique presented in [8]. The second ingredient of the procedure is derived from the observation that the unknown currents can be interpreted as produced by the field diffracted at the array edge, which is excited by the Floquet waves pertinent to the infinite configuration. Following this physical interpretation, the unknown in the IE is efficiently represented by a very small number of basis functions with domain on the entire array aperture. In this framework, the description of the fine details associated to feed points, edge element conditions, etc. are attributed to the solution of the associated infinite array, which has to be added to the fringe solution.

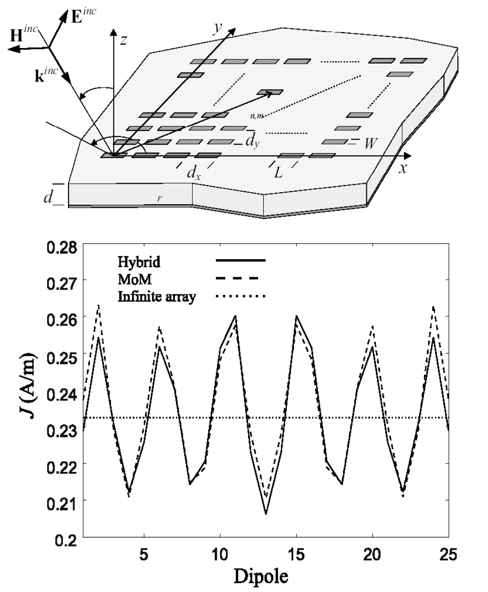

Fig.3 *Finite rectangular periodic array of conducting printed dipoles illuminated by an incident plane wave. Truncated FW results and comparison with full-wave element by element full-wave analysis for the current pattern.*

The hypothesis of linear-phase excitation allows a very simple and efficient solution of the infinite array that is associated with the actual one in terms of Floquet wave (FW) expansions. These FWs are considered as producing diffraction effects and relevant fringe current perturbations at the edge of the array. The key point of the presently described procedure is the treatment of these *unknown* fringe currents in terms of diffracted-ray global basis function expansion. For a grounded slab problem, each FW induces spherical and conical space wave diffracted rays emanate from array corner and edge, respectively; in addition, due to the discontinuity at the boundary between the array region to the bare slab region, surface plane- and cylindrical-waves are excited at edges and corners of the array, respectively. Uniform asymptotic expressions of these contributions are given in [9]. The FW-induced edge surface wave dominate asymptotically the field far from the edge, and constitutes the dominant basis function to be included in the global fringe

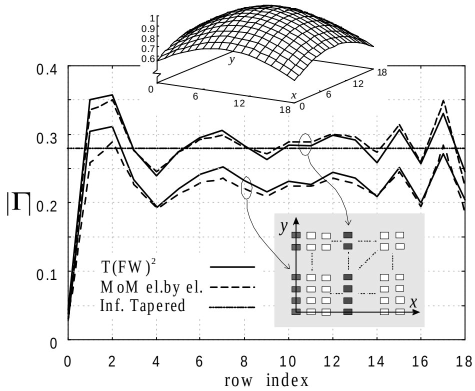

Fig.4. *Magnitude of the active reflection coefficient for an array of 19×19 open-ended waveguides, with 20° E-plane tilted beam and gaussian amplitude excitation (see inset). The waveguide dimensions are a=5714λ b=0.254λ ; periodicities are dx=628λ, dy=0.290λ The 'T(FW)²' label denotes the use of globa diffraction based basis functions. Dashed line denote the conventional element by element approach. The reference flat line is relevant to the solution for the infinite array.*

current expansion.

The coefficients associated to the unknown expansion (diffraction) functions are found by solving via method of moments the pertinent integral equation (IE), derived from the IEs for the finite- and infinite arrays, using as unknown function the difference between the exact solution of the finite array and that of the associated infinite array. The number of diffracted ray basis function which are involved in the process varies from 9 to 16 for rectangular arrays (note that only the dominant FW and the first evanescent are practically important in driving the diffraction process), the final size of the matrix to be inverted is extraordinary small, and, more remarkable, independent on the array size. This obviously imply a tremendous compression rate of the MoM matrix, with a negligible pre-processing time only due to the solution of the infinite array.

The type of array elements that can be studied directly with the this method include slots, cavity-backed apertures, dipoles in free space, and patches. Two examples of application of are shown here, that are concerned with patch array [10] and open ended waveguide arrays [11]. The results in Fig. 3 refers to an array of 25X25 dipoles printed on a grounded substrate (Geometrical and electrical data: dimensions of the dipoles L=0.6 mm; W=0.1 mm; relative permittivity ε_r =10.2; thickness of substrate d=0,1905 mm; periodicity d_x=d_y=0,8 mm, frequency f=7GHz).

Extension to weakly varying tapering was addressed in [8], and treated in [11] with reference to the case of open-ended rectangular waveguide arrays (Fig.4). This extension uses local adiabatic modification of the infinite array solution, enriched by the slope diffraction theories of truncated FW.

4. CONCLUSIONS

Two methods have been addressed here for the compression of the MoM matrix of large arrays composed by planar elements. The first, not restricted to periodic arrays, is based on the definition of local synthetic basis functions defined on sub-blocks of the structures, and generated by solving problems of reduced dimensions. The second technique is applicable to periodic or quasi-periodic arrays with slowly varying amplitude and linear phase excitation; it is based on the definition of global, diffraction-based FW-induced basis functions which are independent on the array dimension. As typical in high-frequency mechanisms, the diffraction based global basis functions are defined on the basis of canonical problems, whose asymptotic treatment is the key point of the method.

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