High-Order Integral Equation Solution for Scattering by Composite Materials

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Introduction

A high-order method of moment solution for the electromagnetic scattering by complex objects with composite materials is presented. The generalized technique can analyze material scattering by homogeneous and inhomogeneous material and conducting objects through both surface and volume equivalent currents. The solution employs high-order basis functions and a quadrature point-based discretization. It is shown that the solution leads to high-order convergence for the electromagnetic scattering by heterogeneous scatterers. This is validated here through the study of canonical scattering structures.

Formulation

Consider the electromagnetic interaction with an inhomogeneous material scatterer made of a composite of penetrable and conducting materials. A surface separating volumes V_i and V_j is denoted as $S_{i,j}$. Let $S_{i,j}^{+}$ denote the surface just inside V_i, and $S_{i,j}^{-}$ denote the surface just inside V_j. Equivalent current densities are then placed on all surfaces separating each material volume. These are defined as:

$$\vec{J}_{i,j}^{+} = \hat{n}_i \times \vec{H}\Big|_{S_{i,j}^{+}}, \quad \vec{M}_{i,j}^{+} = -\hat{n}_i \times \vec{E}\Big|_{S_{i,j}^{+}}, \vec{J}_{i,j}^{-} = \hat{n}_j \times \vec{H}\Big|_{S_{i,j}^{-}}, \quad \vec{M}_{i,j}^{-} = -\hat{n}_j \times \vec{E}\Big|_{S_{i,j}^{-}}, \tag{1}$$

where $\hat{n}_i$ and $\hat{n}_j$ are the unit normal directed into V_i and V_j. Since $\hat{n}_j = -\hat{n}_i$,

$$\vec{J}_{i,j}^{+} = -\vec{J}_{i,j}^{-} = \vec{J}_{i,j}, \quad \vec{M}_{i,j}^{+} = -\vec{M}_{i,j}^{-} = \vec{M}_{i,j}. \tag{2}$$

On the surface of a conductor, only the electric current density is supported. Thus,

$$\vec{J}_{i,p} = \hat{n}_i \times \vec{H}\Big|_{S_{i,p}^{+}}. \tag{3}$$

Each material volume is assigned a *background* material profile $(\varepsilon_{ib}, \mu_{ib})$. If $(\varepsilon_i, \mu_i) \neq (\varepsilon_{ib}, \mu_{ib})$, this results in the need of an equivalent volume current density, expressed as:

$$\vec{J}_{Vi}(\vec{r}) = j\omega\varepsilon_{ib}\left(\frac{\varepsilon_i(\vec{r})}{\varepsilon_{ib}} - 1\right)\vec{E}(\vec{r}), \quad \vec{M}_{Vi}(\vec{r}) = j\omega\mu_{ib}\left(\frac{\mu_i(\vec{r})}{\mu_{ib}} - 1\right)\vec{H}(\vec{r}), \tag{4}$$

where (ε_i, μ_i) are the permittivity and permeability of the physical material medium.

If $(\varepsilon_i, \mu_i) = (\varepsilon_{ib}, \mu_{ib})$, the volume current densities are identically zero. If $(\varepsilon_{ib}, \mu_{ib}) = (\varepsilon_{jb}, \mu_{jb})$, the equivalent surface current density on $S_{i,j}$ is zero.

The surface and volume equivalent current densities in V_i radiate in an equivalent homogeneous material volume of profile $(\varepsilon_{ib}, \mu_{ib})$. The *scattered* field radiated by the equivalent currents in V_i is expressed as:

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$$\vec{E}_i^{scat}\left(\vec{J}_{eq},\vec{M}_{eq}\right)=\eta_{ib}\mathbf{L}_i\left(\vec{J}_{eq}\right)-\mathbf{K}_i\left(\vec{M}_{eq}\right), \tag{5}$$

$$\vec{H}_i^{scat}\left(\vec{J}_{eq},\vec{M}_{eq}\right)=\mathbf{K}_i\left(\vec{J}_{eq}\right)+\eta_{ib}^{-1}\mathbf{L}_i\left(\vec{M}_{eq}\right), \tag{6}$$

where

$$\mathbf{L}_i\left(\vec{X}_{eq}\right)=-jk_{ib}\int_{\Omega}\left[\overline{\overline{I}}+\frac{1}{k_{ib}^2}\nabla\nabla\right]\cdot\vec{X}_{eq}\left(\vec{r}'\right)G_i\left(\vec{r},\vec{r}'\right)d\Omega', \tag{7}$$

$$\mathbf{K}_i\left(\vec{X}_{eq}\right)=\int_{\Omega}\nabla G_i\left(\vec{r},\vec{r}'\right)\times\vec{X}_{eq}\left(\vec{r}'\right)d\Omega', \tag{8}$$

and Ω is either a surface or volume, $\overline{\overline{I}}$ is the unit diad, $G_i\left(\vec{r},\vec{r}'\right)=e^{-jk_{ib}|\vec{r}-\vec{r}'|}/4\pi|\vec{r}-\vec{r}'|$, $k_{ib}=\omega\sqrt{\varepsilon_{ib}\mu_{ib}}$ and $\eta_{ib}=\sqrt{\mu_{ib}/\varepsilon_{ib}}$.

A hybrid surface/volume integral formulation is then derived by enforcing the appropriate constraints on each boundary and/or each volume. A combined field formulation is applied on material surfaces [1, 2] (or the PMCHWT formulation [3]):

$$\hat{n}_i\times\vec{E}_i^{inc}\Big|_{S_{i,j}^+}-\hat{n}_i\times\vec{E}_j^{inc}\Big|_{S_{i,j}^-}=-\hat{n}_i\times\vec{E}_i^{scat}\Big|_{S_{i,j}^+}+\hat{n}_i\times\vec{E}_j^{scat}\Big|_{S_{i,j}^-} \tag{9}$$

$$\hat{n}_i\times\vec{H}_i^{inc}\Big|_{S_{i,j}^+}-\hat{n}_i\times\vec{H}_j^{inc}\Big|_{S_{i,j}^-}=-\hat{n}_i\times\vec{H}_i^{scat}\Big|_{S_{i,j}^+}+\hat{n}_i\times\vec{H}_j^{scat}\Big|_{S_{i,j}^-} \tag{10}$$

where $\vec{E}_i^{inc},\vec{H}_i^{inc}$ are radiated by impressed sources in region *i* and $\vec{E}_i^{scat},\vec{H}_i^{scat}$ are radiated by equivalent currents in volume V_i (similarly for subscript *j*). On the surface of a perfect conductor, a combined field integral operator (CFIE) is applied [4]:

$$\begin{aligned}\frac{\alpha}{\eta_{ib}}\hat{t}\cdot\vec{E}_i^{inc}\Big|_{S_{i,p}}+(1-\alpha)\hat{t}\cdot\hat{n}\times\vec{H}_i^{inc}\Big|_{S_{i,p}}\\=\frac{\alpha}{\eta_{ib}}\left(-\hat{t}\cdot\vec{E}_i^{scat}\Big|_{S_{i,p}}\right)+(1-\alpha)\left(\hat{t}\cdot\vec{J}_{i,p}-\hat{t}\cdot\hat{n}\times\vec{H}_i^{scat}\Big|_{S_{i,p}}\right)\end{aligned} \tag{11}$$

Finally, in regions where the volume currents are non-zero, the constraints applied are:

$$\vec{E}_i^{inc}\left(\vec{r}\right)=\vec{J}_{V_i}\left(\vec{r}\right)\Big/\left(j\omega\varepsilon_{ib}\left(\varepsilon_i(\vec{r})/\varepsilon_{ib}-1\right)\right)-\vec{E}_i^{scat}\left(\vec{r}\right) \tag{12}$$

$$\vec{H}_i^{inc}\left(\vec{r}\right)=\vec{M}_{V_i}\left(\vec{r}\right)\Big/\left(j\omega\mu_{ib}\left(\mu_i(\vec{r})/\mu_{ib}-1\right)\right)-\vec{H}_i^{scat}\left(\vec{r}\right). \tag{13}$$

where $\vec{r}\in V_i$.

Discretization

The volume and surfaces are discretized using curvilinear cells that represent all boundaries and material parameters to high-order accuracy. In this work, curvilinear hexahedron and curvilinear quadrilaterals are used. The basis functions are defined as the product of one-dimensional Legendre polynomials with support limited to the cell. Thus, a set of basis functions complete to polynomial order *p-1* are defined for an individual cell as:

$$\vec{j}_n\left(\vec{r}\right)=\vec{a}_iP_j\left(u^1\right)P_k\left(u^2\right)/\sqrt{g},\ \left\{u^1,u^2\in(0,1);i\in(1,2);\ j,k\in(0,p-1)\right\} \tag{14}$$

for quadrilaterals, and

$$\vec{j}_n\left(\vec{r}\right)=\vec{a}_iP_j\left(u^1\right)P_k\left(u^2\right)P_l\left(u^3\right)/\sqrt{g},\ \left\{u^1,u^2,u^3\in(0,1);i\in(1,3);\ j,l,k\in(0,p-1)\right\} \tag{15}$$

for hexahedra, where, $P_l\left(u\right)$ is the *l*-th order Legendre polynomial mapped over the range of (0,1), and $\vec{a}_i$ are unitary vectors [5].

A quadrature rule is defined for each cell. For quadrilateral and hexahedron, the quadrature rule is conveniently defined via a product rule of one-dimensional Gauss-

Legendre quadratures. At each quadrature point the unitary vectors are used for the test vectors. This leads to a total of $3p^3$ constraints per volume cell and $2p^2$ constraints per surface cell. The method of moment formulation is then derived via point-matching at each quadrature cell with the test vectors, as described in [6-9].

Validation

To validate the above technique, first consider the scattering by a 10 cm radius PEC sphere coated with a lossy dielectric layer. The coating is 1 cm thick and is defined by a relative permittivity $\varepsilon_r = 9 - 0.3j$ at 1 GHz. The surface of the PEC sphere is discretized with 54 fourth-order curvilinear quadrilaterals. A 3×3-point Gauss-Legendre quadrature rule is defined on each patch. Initially, the background material of the layer is chosen to be free space. This results in volume equivalent currents only. To this end, a single layer of 54 fourth-order curvilinear hexahedron with $3\times3\times3$-point quadrature rules were used to represent the volume equivalent currents. Next, the same structure was simulated by choosing the background material $\varepsilon_{rb} = 9 - 0.3j$, leading to surface equivalent currents only. These were represented by 54 fourth-order quadrilateral cells again with 3×3-point Gauss-Legendre quadrature rules defined on each patch. The RCS calculated by both simulations is illustrated in Figs. 1 (a) and (b) for V-V and H-H polarizations. The VIE formulation has a mean error of $\sim 10^{-3}$, whereas, the surface formulation (PMCHWT) has a mean error of $\sim 10^{-2}$.

Next, consider the scattering by a PEC ogive coated with material layers. The PEC ogive had a 0.3 m diameter at the center and a 0.6 m length. The ogive was coated with two homogeneous dielectric coatings with outer dimensions $(0.5\text{ m}, 0.7\text{ m})$ and $(0.7\text{ m}, 0.9\text{ m})$, respectively (referring to center diameter and major axis). For the case presented, the inner layer was free space and the outer layer was a dielectric with a relative permittivity $\varepsilon_r = 4$. The background materials were chosen to be free space, and volume elements were subsequently used to model the material coating and surface elements used to model the surface currents on the PEC (illustrated in Fig. 2). The convergence of the RCS at 300 MHz for various orders and a fixed discretization is illustrated in Fig. 3. The mean deviation of the two higher discretizations is on the order of 0.1 dB.

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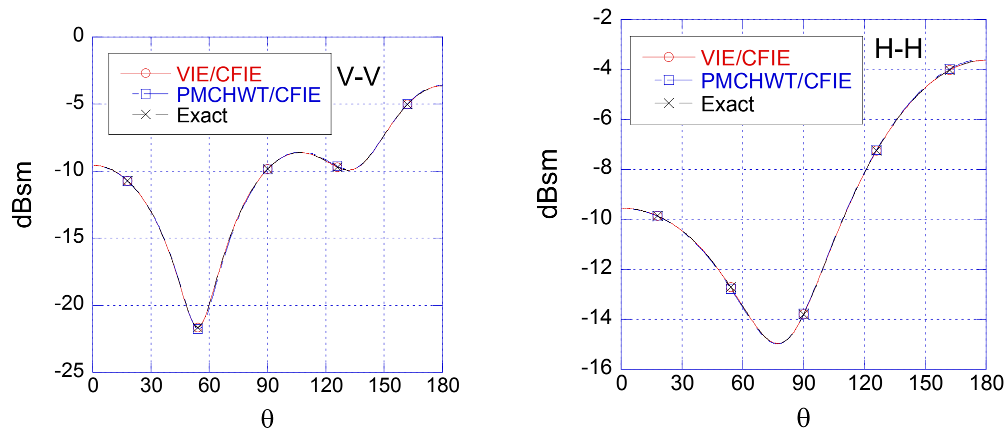

(a) (b)

Fig. 1 Bistatic RCS of the coated PEC sphere at 1 GHz in the $\phi = 0$ plane computed via the hybrid VIE/CFIE and PMCHWT/CFIE methods. (a) V-V, and (b) H-H polarizations.

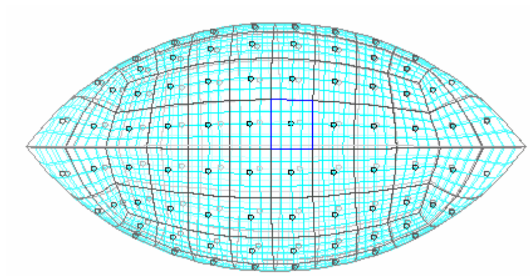

Fig. 2 The surface mesh of the coated ogive illustrating the curvilinear quadrilateral cells.

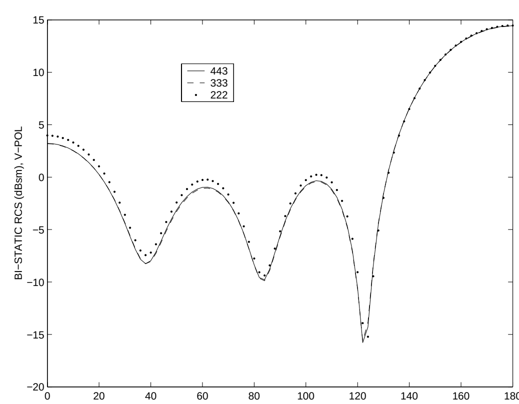

Fig. 3 The RCS of the coated ogive for increasing order. The first two orders are along the transverse directions (surface and volume cells). The third order is along the normal direction for the volume cells.