A sub-cellular algorithm for wire transmission lines in the FDTD method

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1 INTRODUCTION

Three distinct approaches to the problem of representing thin wires in the FDTD method have been identified. The first was introduced by Holland, [1], in 1981 and later extended by Ledfeldt, [2], Edelvik, [3], and Berenger, [4]. This method involves the solution of an extra differential equation which relates the current and charge on the wire to the surrounding tangential electric field. While these methods have been shown to give good results, the formulation lacks rigour, particularly in the definition of the so-called “in cell inductance”. The second was introduced by Umashankar and Taflove et al. [5] in 1987 [6] in 1988 and later extended by Boonzaier, [7], Douglas [8], Makinen, [9] and Bingle [10]. In this approach, the Contour Path integral interpretation of FDTD is used in conjunction with the known singular behaviour of the fields in the vicinity of the wire in order to improve the accuracy. So far, this method is restricted to wires orientated in the direction of the FDTD Cartesian axes. The third approach was introduced by Railton, [11] in 1994 and extended by Craddock [12]. This method is based on the Weighted Residuals interpretation of FDTD in which basis and weighting functions are chosen to well approximate the field behaviour near the wire.

So far, the third approach has been successfully been applied to wire dipoles and transmission lines which are lined up with the Cartesian mesh[13]. In this contribution the extension of this method to wires which are offset from the mesh is described.

2 THEORY

Consider the case where a thin wire is orientated in the z direction. The asymptotic behaviour of the fields are as follows:

$$E_z(r) \propto H_z(r) \propto \ln\left(\frac{r}{a}\right)$$

$$E_x(x,y) \propto H_y(x,y) \propto \frac{x}{x^2+y^2}$$

$$E_y(x,y) \propto H_x(x,y) \propto \frac{y}{x^2+y^2}$$

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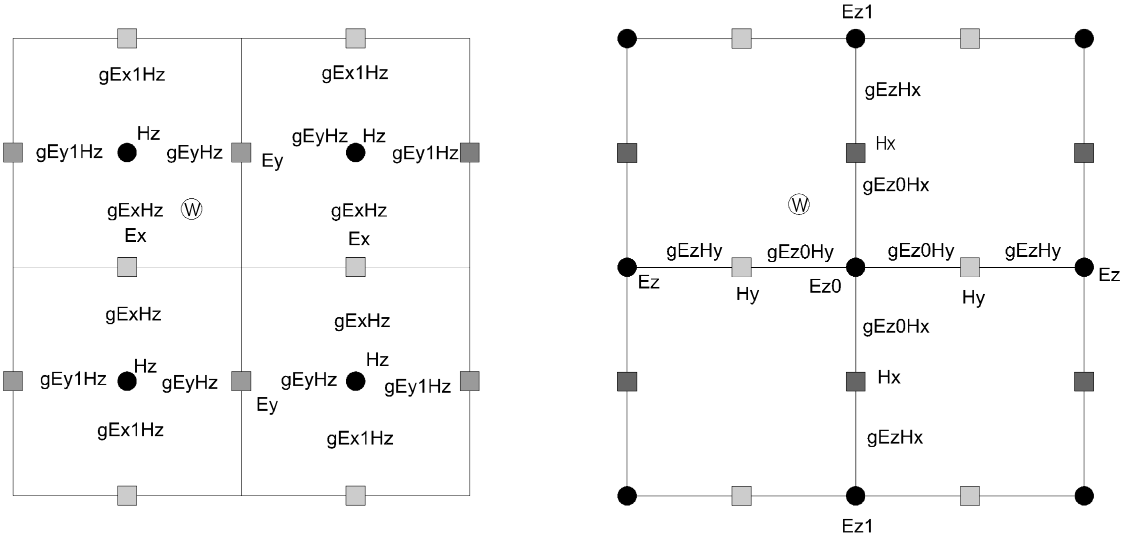
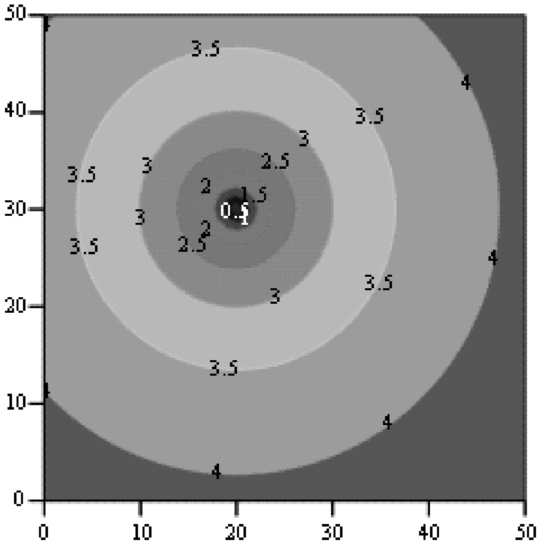


Figure 1 - Equivalent circuit components for a thin wire – TM and TE planes

These functions, truncated to give the same support as the original pulse basis functions, can be used as basis functions for the nodes immediately adjacent to the wire. In Figure 1, the wire, shown as a "W" in a circle, and the affected nodes are shown. An equivalent circuit capacitor is associated with each node and an equivalent circuit gyrator is associated with each pair of interacting nodes [14]. The values of these equivalent circuit components are calculated from the basis and weighting functions chosen for each node. An example of the E_{z0} basis function for the case shown in Figure 1 is plotted on the right. Details of the calculation of the values of the equivalent components are given in [13].



Ez0_basis

3 RESULTS FOR A TWO WIRE TRANSMISSION LINE

As an example of the use of the thin wire algorithm, the case of a transmission line formed by a wire over a ground plane is considered. This is formally the same as a twin wire transmission line. This example, which appears less in the literature than isolated wires, such as dipoles and loops, has properties which make it susceptible to different types of approximation error. In particular, the fields are much less symmetrical and also, different parameters, such as characteristic impedance, are of interest.

Results for the characteristic impedance of a wire 10mm above a ground plane and with radii varying from 0.05mm to 0.5mm are presented in Figure 2. Here, the results are compared with analytical formulae as well as results obtained using the methods of [9] and [15]. The mesh size used was 2mm in all directions. It can be seen that although the method described in [9] gives results which are, on average, much more accurate than those from basic FDTD, there is still a discrepancy. The method of [15] produces results which are not very different from those of the basic FDTD method. This would be expected since the behaviour of the wire, especially when the radius is small, is dominated by the singular behaviour of the field near the wire. This effect is not taken into account in [15].

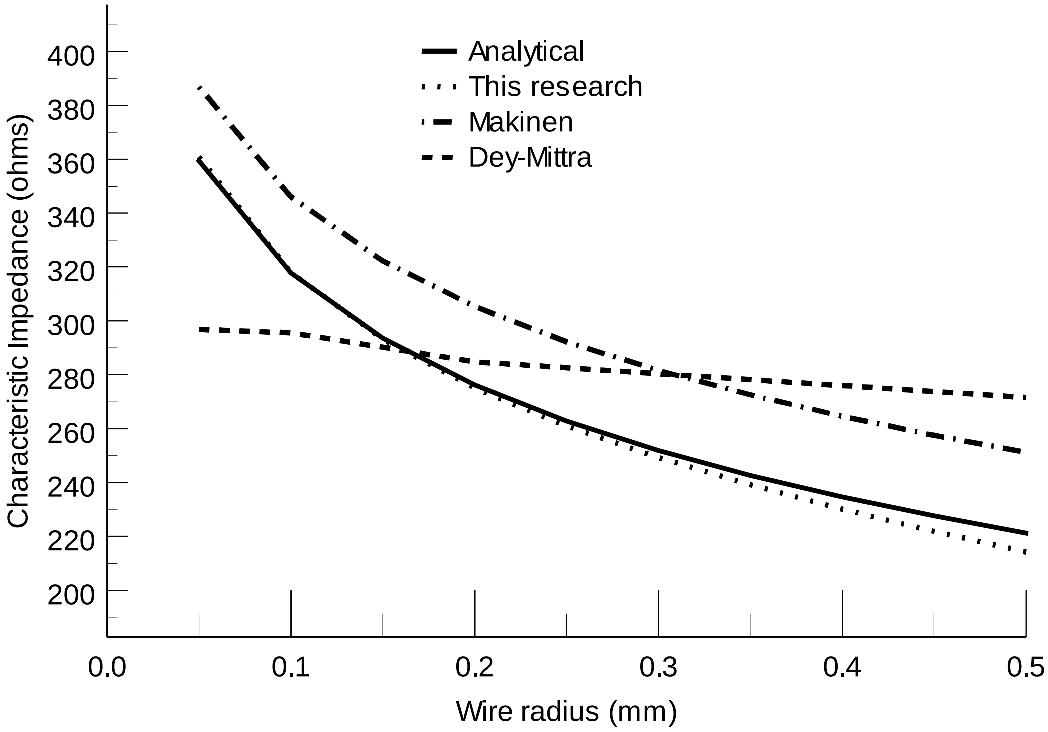


Figure 2 - Characteristic impedance of a two wire transmission line

4 CONCLUSION

In this contribution the application of a sub-cellular algorithm, based on the weighted residules interpretation of FDTD, to wire transmission lines has been described. The method is shown capable of giving results which are in good agreement with analytical formulae. This approach is readily extended to other types of transmission line, such as microstrip, by choosing the appropriate basis functions.

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