On Grid Subdivisions for the Simplicial Discretization of Maxwell's Equations

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During the discretization of the first-order pair of Maxwell's curl equations on irregular and simplicial (triangular in 2-D, tetrahedral in 3-D) grids, one is confronted with the inherent asymmetry caused by the fact (I) that a dual grid of a simplicial primal grid is not simplicial anymore. Accordingly, the use of dual staggered grids in this case does not retain the symmetry of the hexahedral (cellular) geometry, which impacts the consistent discretization of the first-order curl equations [1]-[3].

In the differential forms framework [1]-[9], each *n*-cell element (node, edges, facets, cells) of the grid is mapped (a bijection) to a basis element on the *n*-cochain space (discrete fields), or equivalently, the space of discrete differential *n*-forms. If one chooses to associate the 1-form E and the 2-form B with the primal grid, then the 1-form H and the 2-form D become necessarily associated with the dual grid. In the discretization of the metric part of Maxwell's equations (Hodge operators) [4], it is necessary to employ an interpolation (continuum representation) of the discrete fields. The interpolants are called *Whitney forms* (for 1-forms in the Euclidean space, they just correspond to the usual edge elements of the finite element literature) [3],[5],[6] which are defined for a simplicial grid only (II). Note that edge elements for non-simplicial grids do exist, but they are not divergence-free interpolants anymore. As a result of (I) and (II), the discretization of the metric part of Maxwell's equations becomes ambiguous.

Here, we discuss the use of grid (lattice) subdivisions to restore the symmetry between the primal and dual grid in simplicial discretizations of the first-order Maxwell's curl equations and allow for the definition of Whitney interpolants in both grids. After such subdivisions, the full discretization can be realized in the (simplicial) first subdivision grid and the symmetry between the fields in the primal and dual grids is fully restored. We discuss the use of both barycentric [7] and standard [8] subdivisions for this purpose.

References

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