Data Structures for Computational Electromagnetics Inherited from Algebraic Topology

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In both finite element analysis and the study of manifolds, the notion of a simplicial complex has long been used as a basic data structure enabling one to model spaces without making implicit geometric or topological assumptions. In electromagnetics, the basic equations are most naturally stated in terms of differential forms. That is, Maxwell's equations are integral laws and information about the electromagnetic field is obtained by integrating or appealing to the generalized form of Stokes' theorem to deduce differential versions of Maxwell' equations and interface conditions. In the last two decades, within computational electromagnetics, Whitney forms have emerged as an ideal solution to the quest for a discrete model of differential forms phrased in terms of the data structures of simplicial complexes (see, for example A.Bossavit, Computational Electromagnetism, Academic Press Boston, 1998, or P.W. Gross and P.R. Kotiuga, Data Structures for Geometric and Topological aspects of Finite Element Algorithms, Ch. 6 of Geometric Methods for Computational Electromagnetics, PIER Vol.32, EMW Publishing, 2001). The adoption of Whitney forms has resolved many outstanding problems in vector interpolation and riddles such as the computation of spurious modes in cavities.

- Given the success of simplicial data structures and Whitney forms, we examine the roots of simplicial techniques and geometric integration theory within topology, outgrowths, and inevitable technology transfer to computational electromagnetics. Specifically, we will examine the following mathematical milestones in the context of open problems in, and data structures for, 3-d computational electromagnetics:

- De Rham's 1931 thesis and simplicial techniques in developing homology theory in the 1920's. (i.e. first proofs of many statements in Ch.1 of Maxwell's treatise).
- The appearance in the 1940's of Eilenberg-Steenrod axioms for a cohomology theory.
- Reaction to axiomatics; Andre Weil's paper (A. Weil, Sur les Theorems de de Rham, Commentarii Matematicae Helvetici, 1952), developing the formula for Whitney forms, and Whitney's subsequent geometric integration theory.
- Milnor's theorem on CW complexes and the rejection of triangulations by the mainstream of algebraic topology in the late 1950's.
- Subsequent uses of Whitney forms in rational homotopy theory (P.A. Griffiths and J.W. Morgan, Rational Homotopy Theory and Differential Forms, Progress in Math, Vol. 16, Birkhauser Boston, 1981), combinatiorial Hodge theory and the understanding of torsion invariants (W. Muller, Analytic Torsion and R-Torsion of Riemannian Manifolds, Advances in Math., Vol.28, pp 233-305, 1978).
- Semi-simplicial objects in algebraic topology (B. Gray, Bull. AMS, (38)2, 217-220, Review of: Simplicial Homotopy Theory, P. G. Goerss and J .F. Jardine, Progress in Math, vol.174, Birkhauser, Basel, 1999) as a combinatorial abstraction of space.

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