Stable Space-Time Mortar Finite Element Approach for FDTD Subgridding

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The conventional Finite Difference Time Domain (FDTD) scheme is widely used because of its ability to tackle large and complex electromagnetic systems in various applications such as antennas, waveguiding structures, electromagnetic compatibility and biological interactions with human bodies. This powerful feature comes from the very nature of its mesh which is regular and orthogonal. The algorithm is explicit and avoids a linear system solving which occurs for the standard time domain version of the Finite Element Method (FEM). However this kind of mesh could be not suitable to model complex geometries which are not aligned to grid points. Thus, many approaches have been investigated to overcome the problem : how to model arbitrary material properties and geometry shapes while preserving the efficiency of the FDTD ? Among them, subgridding techniques have been developed with focus on the spatial connection or interpolation schemes at the interface and the time stability. In this work, the subgridding in FDTD is derived from a FEM formulation in 3D in the framework of Whitney elements and by knowing that the FDTD can be derived from it using mass lumping techniques. Mortar finite elements are used to glue the non-conforming meshes, a coarse grid and a fine one, in space. The procedure is equivalent to a FEM formulation with a Lagrange multiplier. Applied to cubic grids, the subgridding scheme in space is obtained using mass lumping techniques. Analogy with network equations naturally provided by Whitney elements will be shown. The connection scheme of the problem discretized in space is reciprocal which means passivity in an equivalent circuits model and thus ensures its stability. The subgridding is also performed in time using the usual leap-frog schemes in each mesh domain. The time stepping scheme is derived by verifying a discrete energy conservation which ensures the stability of the fully discretized problem in space and time. The results for a subgridding factor of 3 will be presented. The presented technique is general and can be applied to match different kinds of non-conforming meshes like FDTD/FEM, FEM/FEM or conforming FEM but with different time steps.