Enhanced transmission through periodic sub-wavelength hole structures

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Introduction

In the past, several research groups have observed, both numerically and experimentally, unusually high *optical* transmission through metal films loaded with periodic arrays of small holes [1]. This enhanced transmission was attributed to the interaction of the incident field with surface plasmon polaritons (SPPs) [1-2], viz. surface waves on the boundary of a half-space with negative permittivity. In the optical regime, several metals indeed support SPPs [2]. In contrast, in the RF regime, metals can be safely modeled as electric conductors with or without losses and do not support SPPs. Thin perfect electric conductor (PEC) plates loaded with regular arrays of small (subwavelength) holes have small transmission coefficients [3]. To the best of our knowledge, no enhanced microwave transmission through such plates has been reported in the open literature. The goal of this study is to formulate conditions under which enhanced transmission through periodically perforated PEC plates nonetheless may be achieved.

"Two period" array of small holes

As an example of a perforated PEC plate that allows for enhanced transmission, consider the "two-period" structure depicted in Figure 1. It comprises of wide bands of doubly- and equi-periodic, orthogonal arrays of small square holes that are separated by narrow strips of solid PEC. The small holes have side length s and periodicity a, with $s < a \ll \lambda$ (λ is the wavelength).

Figure 1: Problem configuration

The PEC strips have width $L-d$ and periodicity L. In what follows, a and L are referred to as the small and large period, respectively. This "two-period" array is excited by a TM_z plane wave with transverse wavenumber k_{x0} and magnetic field along the PEC strips. Enhanced transmission is possible for certain ranges of a, d, L, and k_{x0}.

The proposed structure is motivated by the following observations. It is known that a PEC plate loaded with a dense and regular doubly periodic array of small holes can support a TM_z surface wave (SW) [4]; it follows that such a plate can be modeled accurately by an inductive impedance surface [3]. The purpose of the small period structure in the above "two-period" grating therefore is to support a SW. The large-period grating has dual functionality: (***i***) it couples the incident field into the SW on the small-period structure and (***ii***) subsequently provides a mechanism for diffraction and re-radiation of the SW into the zeroth-order transmitted mode. The SW therefore acts as an "agent" that facilitates transformation of the incident
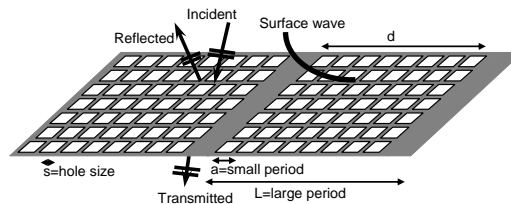
into the transmitted field. This process can lead to full transmittance for a narrow range of frequencies, as discussed below.

Simplified model - periodic impedance surface

The above "two-period" structure can be modelled by a periodic impedance surface that is obtained after replacing the doubly-periodic bands of square holes by an impedance surface. This simplified structure therefore comprises of a singly-periodic array of period L of wide surface impedance strips of width d that are separated by narrow PEC strips of width $L-d$. This structure can be analyzed via conventional expansion of the fields over the spatial harmonics with appropriate boundary conditions on the surface impedance and PEC strips [5]. Doing so leads to the matrix equation for the diffraction coefficients

$$\mathbf{T} = \left[\underline{\underline{\mathbf{I}}} + \tfrac{1}{2}Y_s\underline{\underline{\mathbf{Z}}}\,\underline{\underline{\mathbf{P}}}\right]\mathbf{A}, \quad \mathbf{\Gamma} = \mathbf{A} - \mathbf{T} \tag{1a}$$

$$\mathbf{T}, \mathbf{\Gamma} = \{T_n, \Gamma_n\}|_{n=-\infty}^{\infty}, \quad \mathbf{A} = \{\delta_{0n}\}|_{n=-\infty}^{\infty} \tag{1b}$$

$$\underline{\underline{\mathbf{P}}} = \left\{ P_{m,n} = L^{-1}\int_{-d/2}^{d/2} e^{j(k_{xm}-k_{xn})x}dx = (d/L)\mathrm{sinc}\left[(k_{xm}-k_{xn})d/2\right] \right\}\Big|_{n=-\infty}^{\infty} \tag{1c}$$

$$\underline{\underline{\mathbf{Z}}} = \{Z_n\delta_{mn}\}|_{m,n=-\infty}^{\infty}, \; Z_n = \eta\frac{k_{zn}}{k_0}; \quad k_{zn} = \sqrt{k_0^2 - k_{xn}^2}, \quad k_{xn} = k_{x0} + \frac{2\pi n}{L}. \tag{1d}$$

Here $\eta = 120\pi$ and $k_0 = 2\pi/\lambda$ is the free space impedance and wavenumber, respectively; Y_s is the surface admittance; T_n and Γ_n are the nth harmonic diffraction (transmission and reflection) coefficients; $P_{m,n}$ is the coupling matrix; and Z_n and k_{xn} are the characteristic impedance and wavenumber of the nth spatial harmonic, respectively. Although Eqs. (1) provide a complete solution to the problem, for design purposes, it would be to our benefit to also have explicit and physically transparent expressions for the transmission coefficients and the locations of its maxima and minima. To this end we only retain the zeroth and first harmonics in equations (1). This is allowed since these harmonics are excited much more strongly than all others when $k_{x1} \approx k_p$ (k_p being the surface wave wavenumber). For simplicity we also assume a normal incidence $k_{x0} = 0$. Then the approximated transmission coefficient (denoted by $\widetilde{T}_0$) is given by

$$\widetilde{T}_0 = 1 - \frac{1 + 2Z_sY_1\tilde{P}_{1,1}}{[1 + 2Z_sY_0P_{0,0}][1 + 2Z_sY_1\tilde{P}_{1,1}] - 4Z_s^2Y_1Y_0P_{1,0}\tilde{P}_{0,1}}, \tag{2}$$

where $\tilde{P}_{1,1} = P_{1,1} + P_{1,-1}$ and $\tilde{P}_{0,1} = P_{0,1} + P_{0,-1}$; $Y_n = Z_n^{-1}$ is the nth harmonic characteristic admittance; $Z_s = Y_s^{-1}$ is the surface impedance. Solving (2) for L when $\widetilde{T}_0 = 1$ and $\widetilde{T}_0 = 0$, the conditions for the "full" and "zero" transmission can be readily found

$$\left(\frac{L}{\lambda}\right)^{\mathrm{full}} = \left[1 - \left(\frac{2\tilde{P}_{1,1}}{Y_s\eta}\right)^2\right]^{-1/2}, \; \left(\frac{L}{\lambda}\right)^{\mathrm{zero}} = \left[1 - \left(\frac{2\tilde{P}_{1,1}}{Y_s\eta}\right)^2\left(1 - \frac{\tilde{P}_{0,1}P_{1,0}}{P_{0,0}P_{1,1}}\right)^2\right]^{-1/2}. \tag{3}$$

Note that these phenomena occur for values of L around $\frac{L}{\lambda} = \frac{k_0}{k_p} = [1 - \left(\frac{2}{Y_s\eta}\right)^2]^{-1/2}$, where k_p is the SW wavenumber, indicating the importance of the SW. However, mathematically, due to the fact that $\tilde{P}_{1,1} < 1$ and, physically, due to interaction of the SW and the incident field, these locations are shifted.

Numerical examples

We first verify the existence of the SWs. Consider the structure in Figure 1 where we leave only the doubly periodic array of small period a. We choose $s = 3mm$ and $a = 4.5mm$. We also choose the wavelength $\lambda = 30mm$ (i.e. $\lambda/s = 10$). We then calculate the TM_z transmission coefficient $T_{0,0}$ via the method of moments with periodic Green's function [6]. The corresponding surface impedance can be calculated then via the transmission line model.

Figure 2(a) shows the real and imaginary parts of $T_{0,0}$ as a function of k_{x0} for both the propagating ($k_{x0} \leq k_0$) and evanescent ($k_{x0} > k_0$) spectra. We find that around $k_{x0}/k_0 \approx 1.01$ the magnitude of $T_{0,0}(k_{x0})$ exhibits a pole type behavior that is an evidence of the SW existence. Figure 2(b) depicts the corresponding surface impedance Z_s which is inductive since $\text{Im}Z_s > 0$.

We then consider the "two period" structure in Figure 1 where the hole size s and the small period a are the same as in the previous example and the large period is chosen $L = 31.5mm$ (i.e. each eighth hole is replaced with the PEC). Figure 3(a) shows the zero order transmission coefficient as a function of the ratio L/λ. The same figure shows the similar results for the periodic impedance surface with period L and surface impedance $Z_s = 10.5\Omega$. We give both the "exact" result calculated via (1) and the approximate result calculated via (2).

We indeed observe the "full" and "zero" transmission in Fig. 3(a) as predicted. We find also that the results for the "two period" structure are similar to those for the periodic impedance surface model. However in the former case the transmission is weaker and wider that can be explained by stronger interaction with higher order modes. The exact results via (1) for the periodic surface matches well the approximation (2). Note that the locations of the maximum and minimum are close to the ratio $\frac{L}{\lambda} = 0.9985$ corresponding to the SW but they are shifted right.

Finally in Figure 3(b) we show the excitation coefficients of the first order mode and find that the location of its maximum matches the location of the "full" transmission in Figure 3(a). Again the results for the "two period" structure and the periodic impedance surface are similar.

References

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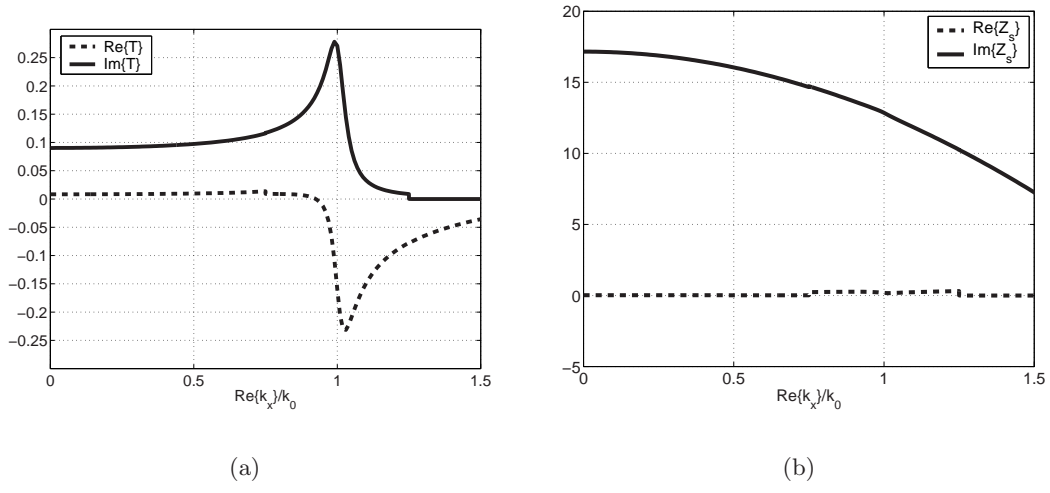
(a)   (b)

Figure 2: Simulation for doubly periodic structure with small period a: (a) Reflection coefficient as a function of k_{x0}; (b) Corresponding surface impedance
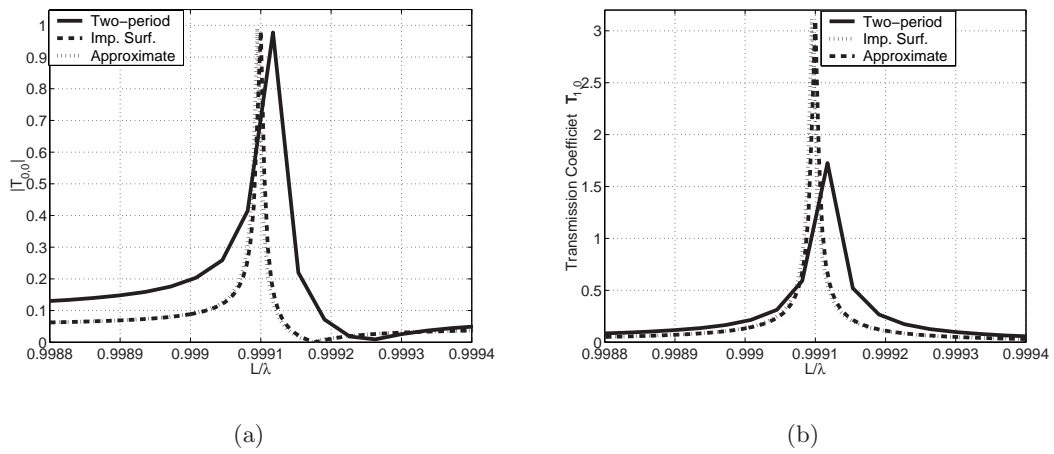

(a)   (b)

Figure 3: Transmission coefficients (a) $T_{0,0}$ and (b) $T_{1,0}$ as a function of the ratio L/λ for $L = 31.5mm$