Analysis of Curved Frequency Selective Surfaces Using the Hybrid Volume-Surface Integral Equation Approach[1]

Chun Yu* and Cai-Cheng Lu

Department of Electrical and Computer Engineering
University of Kentucky
Lexington, KY 40506
E-mail: cclu@engr.uky.edu

1. Introduction

Most of the frequency selective surface (FSS) is made of dielectric substrates and conducting patches that are periodically located on top of the substrates. Because of the many important applications in microwave and optical engineering, it has attracted the attention of investigators, and many methods have been developed to analyze different types of FSS structures [1-3]. For periodic FSS geometries of flat and infinite extent, the most effective method is the conventional method of moments based on Floquet's theorem. However, this method does not apply when the FSS structures are of finite extent, or of finite radius of curvature. In this case, full-wave numerical methods must be employed to determine the scattering and radiation characteristics. Though, the flat-surface FSS is relatively easier to analyze and design, its application is limited. For example, if one wants to filter out unwanted microwave penetration of radome to the antenna compartment in an aircraft, then it is desirable to put FSS on the radome. In this case, the FSS has to be curved with finite extent.

In this paper, we apply the hybrid volume-surface integral equation (VSIE) approach for the analysis of finite and curved FSS structures. The VSIE technique has previously been applied for analysis of the scattering by composite metallic and material targets and input impedance of the printed structures with finite ground plane [4-6]. Compared with other simulation approaches, the VSIE approach presents a number of advantages. For example, it gives flexibility to model structures of real size and of non-flat, arbitrarily shaped conducting structures or dielectrics. Because this method is based on the method of moment solution of the hybrid integral equation, it has high solution accuracy. Moreover, in the VSIE approach, the only Green's function used in the integral operators is the free-space dyadic Green's function, hence, the multilevel fast multipole algorithm (MLFMA) can be easily applied to reduce the computational complexity so that FSS structures of large size and arbitrary shape can be simulated by this method.

2. Description of the VSIE Approach

For a finite extend but flat FSS structure, the most effective method is to use the CG-FFT method in which, the fast Fourier transform is applied to speed up the matrix-vector multiplication in the conjugate gradient iterations. Theoretically, this approach can still be applied if the structure is curved. In this case, the grid points are no longer on the structured grid and pre-corrections are needed to project the physical grid points to the structured grid points so that the FFT can be used. However, one should note that the efficiency of this method is reduced due to the inclusion of extra grid points. The smaller the curvature of the FSS, the more reduction in

---

[1] This work was supported in part by an NSF CAREER award under award No. ECS-0093692, and in part by a Young Investigator Program award from the Office of Naval Research under award No. N00014-00-1-0605.

computation efficiency for the CGFFT. The VSIE algorithm, on the other hand, does not have this limitation. In fact, it does not introduce extra grid points to model the curved structures. Instead, it assigns basis functions on the surface and the dielectric volume only. The basic idea of this approach is to use the surface equivalent theorem and volume equivalent principle to replace the surfaces with surface currents, and the dielectric volume with the volume current [4-6]. Together, the two currents satisfy two simultaneous integral equations: the surface integral equation and the volume integral equation. The two equations are solved simultaneously by the method of moments. To solve the integral equation numerically, the conducting surfaces (ground plane and printed structures are modeled by quadrilateral surface mesh, and the dielectric substrates by the volume hexahedron mesh. Because of the quadrilateral patch, this method can even be applied to model the printed patches that are of arbitrary shape such as circular discs and rings.

Consider a FSS structure in free-space. Let S denotes the collection of all the conducting patches, and V denote the dielectric substrates with permittivity ε and permeability μ. Under external radio wave illumination, a surface current $\mathbf{J}_S$ will be induced on the conducting surface and a volume electric current $\mathbf{J}_V$ and magnetic current $\mathbf{M}_V$ will be induced in the volume. By the equivalence principle, there two current radiates into free-space to form the scattered field. The scattered field is related to the induced equivalent surface and volume currents by

$$\mathbf{E}^{\mathrm{sca}}(\mathbf{r}) = i\omega\mu_0 \int_S \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}_S(\mathbf{r}')\,dS' + i\omega\mu_0 \int_V \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}_V(\mathbf{r}')\,dV' - \nabla \times \int_V \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}_V(\mathbf{r}')\,dV'$$

$$\mathbf{H}^{\mathrm{sca}}(\mathbf{r}) = \nabla \times \int_S \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}_S(\mathbf{r}')\,dS' + \nabla \times \int_V \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}_V(\mathbf{r}')\,dV' + i\omega\varepsilon_0 \int_V \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}_V(\mathbf{r}')\,dV'$$

where, $\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = (\bar{\mathbf{I}} + k_0^{-2}\nabla\nabla)\exp\{ik_0\,|\mathbf{r}-\mathbf{r}'|\}/(4\pi\,|\mathbf{r}-\mathbf{r}'|)$ is the three-dimensional free-space dyadic Green's function, k_0 is the wave number in free-space, $\bar{\mathbf{I}}$ is the unit dyad. If the incident electric field and magnetic field are $\mathbf{E}^{\mathrm{inc}}$ and $\mathbf{H}^{\mathrm{inc}}$, respectively, then the surface current and the volume current satisfy the hybrid volume and surface integral equations that are formally written as follows,

$$\mathbf{E}_{\tan}^{\mathrm{sca}}(\mathbf{r}) = -\mathbf{E}_{\tan}^{\mathrm{inc}}(\mathbf{r}), \quad \mathbf{r} \in S,$$
$$\mathbf{E}^{\mathrm{tot}}(\mathbf{r}) = \mathbf{E}^{\mathrm{sca}}(\mathbf{r}) + \mathbf{E}^{\mathrm{inc}}(\mathbf{r}), \quad \mathbf{r} \in V$$
$$\mathbf{H}^{\mathrm{tot}}(\mathbf{r}) = \mathbf{H}^{\mathrm{sca}}(\mathbf{r}) + \mathbf{H}^{\mathrm{inc}}(\mathbf{r}), \quad \mathbf{r} \in V$$

Since $\mathbf{J}_V = i\omega(\varepsilon_0 - \varepsilon)\mathbf{E}^{\mathrm{tot}}$, and $\mathbf{M}_V = i\omega(\mu - \mu_0)\mathbf{H}^{\mathrm{tot}}$, the above three equations actually contains three unknown vector functions. We use the method of moments to convert the integral equations into a matrix equation. The basis functions used are the roof-top basis functions (for both the surface mesh and the volume mesh). The matrix equations are then solved by either a direct solver or an iterative solver. If iterative solver is used, the matrix-vector multiplication for each iteration step is conducted by the aid of the multilevel fast multipole algoritm, and hence, large scale problems can be simulated. Once the surface electric current on the conducting and volume equivalent current $\mathbf{J}_V$ in the dielectrics are found, the transmission and reflection properties of FSS can be calculated.

3. Numerical Example

In this section, we present two numerical examples to demonstrate the application of the VSIE technique for analyzing the curved and finite-extent FSS structures. The first structure is a truncated dielectric backed FSS that consists of short printed dipoles on a layer of dielectric substrate. The substrate is curved so that the radius of curvature is 21 mm. There are 49 parallel dipoles arranged by 7×7 on top of the substrate, as shown in Figure 1. If the structure is expanded to a flat surface, then each dipole has width W=1.75mm, length L=4.15mm, and is centered on a substrate unit of dimension $a = b = 6\,\text{mm}$. Figure 1 also shows the top view of one unit of the structure. The substrate is electric dielectric with thickness $d = 0.037\,\text{mm}$ and relative permittivity $\varepsilon_r = 3$. An incident plane wave at various frequencies is assumed as the excitation. The incident angles are $(\theta_i, \phi_i) = (0^0, 0^0)$. Figure 2 and 3 shows the calculated forward and backward scattering cross section of the structure for V and H polarized incident waves, respectively. It is interesting to note that there are two frequencies that have minimum backscattering, one at 29 GHz, and the other at 35 GHz. This means that this curved FSS has a dual-frequency band filter property.

Figure 4 and Figure 5 shows the scattering results of the second example. This is a planar FSS of finite extent. The structure is the same as the curved FSS except that a 5×5 dipole array is used. This planar FSS structure is also presented in [3]. The incident plane wave at the incidence angle of $(\theta_i, \phi_i) = (30^0, 0^0)$ is assumed and both vertical and horizontal polarized waves are considered. Figure 4 shows the RCS for vertically polarized incident case and Figure 5 is for the horizontally polarized case. As shown in the two figures, this dielectric backed FSS of finite extent presents good frequency selective characteristics.

4. Conclusion

The hybrid volume-surface integral equation (VSIE) approach has been applied to investigate the curved FSS with the dielectric substrates of finite extent. The scattering RCS of a curved structure and a planar finite periodic structure have been presented. An important advantage of the VSIE algorithm is that it can be used to simulate the finite and arbitrary curved structure. When the finite structure size increases, the multilevel fast multipole algorithm can be applied to reduce the computational complexity.

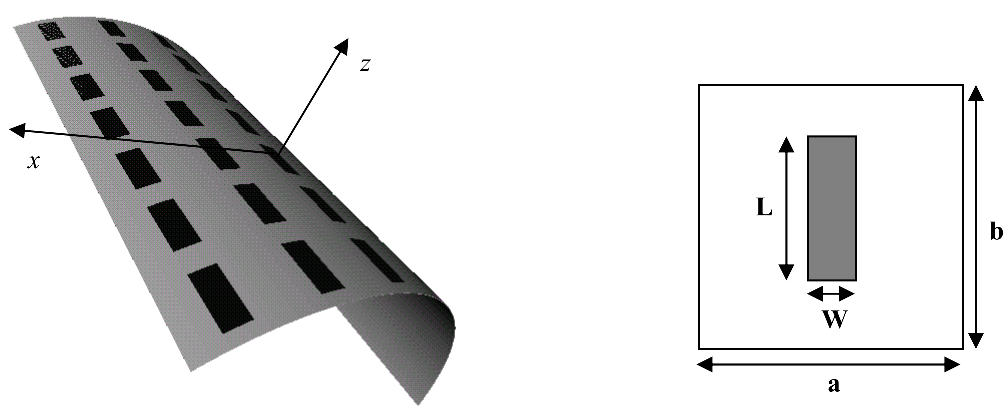

Figure 1. The geometry of the curved FSS structure (left) and one element (right).

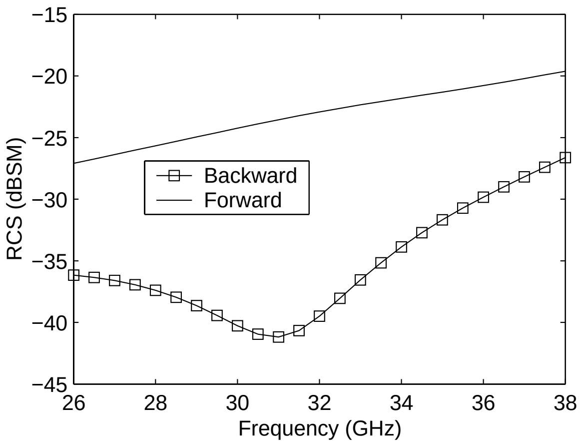

Figure 2. *Calculated RCS of the curved FSS with* 7×7 *elements (V-V-polarization). The plane wave incident direction is in* $-\hat{z}$.

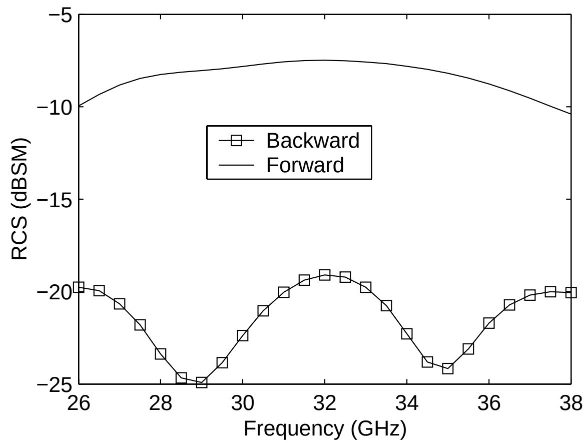

Figure 3. *Calculated RCS of the curved FSS with* 7×7 *elements (H-H-polarization). The plane wave incident direction is in* $-\hat{z}$.

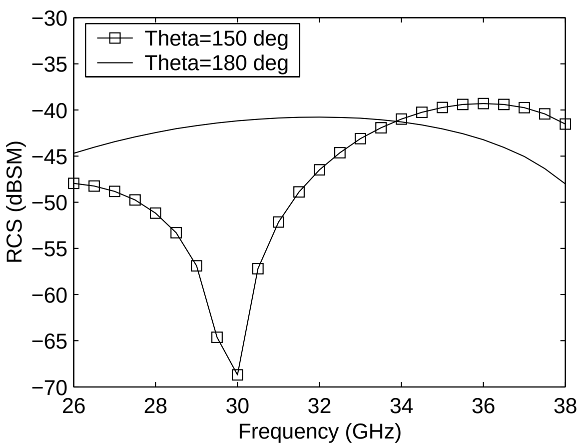

Figure 4. *RCS (V-V Pol.) of the flat FSS with* 5×5 *elements (*$\theta_i = 30^0$, $\phi_i = 0^0$*).*

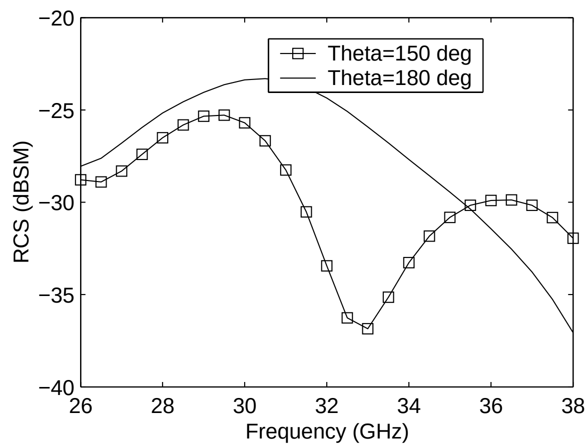

Figure 5. *RCS (H-H Pol.) of the flat FSS with* 5×5 *elements (*$\theta_i = 30^0$, $\phi_i = 0^0$*).*

5. References:

[1] R. Mittra, C. H. Chan and T. Cwik, "Techniques for analyzing frequency selective surfaces-A review", *Proc. IEEE*, vol.76, pp.1593-1615, 1988

[2] R. Mittra and V. V. S. Prakash, "Analysis of large finite frequency selective surfaces embedded in dielectric layers", *IEEE Antennas Propagat. Symposium*, pp.572-575, 2002

[3] J. C. Vardaxoglou, *Frequency selective surfaces*, John Wiley & Sons, New York, 1997.

[4] C. C. Lu and W. C. Chew, "Electromagnetic scattering from material coated PEC objects: a hybrid volume and surface integral equation approach," *The 1999 IEEE Antennas and Propagation Society International Symposium*, Orlando, Florida, July11-16, 1999.

[5] C. C. Lu and W. C. Chew, "A Coupled Surface-Volume Integral Equation Approach for the Calculation of Electromagnetic Scattering from Composite Metallic and Material Targets," *IEEE Trans. Antennas Propagat.*, vol. 48, no. 12, pp. 1866-1868, Dec. 2000.

[6] C. C. Lu and C. Yu, "Analysis of microstrip structures of finite ground plane using the hybrid volume-surface integral equation approach", *The 2002 IEEE Antennas and Propagation Society International Symposium*, pp.162-165, July 2002.