Analysis of FSS Composites Comprising of Multiple FSS Screens of Unequal Periodicity

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1. INTRODUCTION

The problem of analyzing Frequency Selective Surfaces (FSS) has been extensively studied by a number of authors [1, 2], both for single and multiple screens, which are assumed to have identical periodicities but possibly different element configurations. However, certain designs of the FSS composite may involve screens with unequal periodicities, that may be either commensurate or non-commensurate. These systems have not been analyzed in the past, except for the relatively simple case of two widely separated screens with little coupling between them. Dissimilar FSS screens have been analyzed recently by Prakash and Mittra [3] by using an iterative approach valid for small-to-moderate levels of coupling. In this paper we present a technique for analyzing similar FSS systems that may be closely coupled.

To analyze FSSs with unequal periods using the Method of Moments (MoM), we may follow one of the following two approaches. In the first approach (direct method), a global period is identified for the composite as a first step (assuming the periods are commensurate), and the multiple screen problem is then solved in its entirety using a relatively large number of unknowns. An alternative (cascading method) is to derive the scattering characteristics of each of the subsystems individually, and then cascade the scattering matrices, thus obtained, to construct the corresponding matrix for the composite system whose global period is also identified as in the previous case. The difficulty with the first approach is that the problem size often becomes large relatively quickly in terms of the number of unknowns, because the global unit-cell typically includes several unit-cells of each of the individual screens. Furthermore, if the problem is handled by using an iterative procedure based on the Conjugate Gradient Fast Fourier Transform (CGFFT) method, as is often the case for such large problems, then the global unit cells must be discretized by using a relatively large power of two in order ensure that the unit cells of the individual screens are modeled with sufficient geometrical fidelity.

In contrast to the direct method, the cascading technique is not only more efficient, but is often the only viable approach for a system with a large global period. Earlier works have focused on commensurate systems where it is not difficult to find a common global period [2]. The only non-commensurate system that has been analyzed was for the case where the spacing between the FSS screens is large, and only the zero-*th* mode needed to be included in the cascading process [2]. In this paper, we present a technique that extends the analysis to non-commensurate systems for which the spacing is not necessarily large and, hence, the higher-order Floquet harmonics cannot be neglected.

2. CASCADING TECHNIQUE

2.1 CONSTRUCTION OF THE GLOBAL SYSTEM

The geometry of the FSS composite, comprising of N screens is shown in Fig. 1. The period of each individual screen is T_i, with i=1, …, N. Our first step is to identify a global period T_g for the composite structure so that we can analyze it with a single set of Floquet harmonics.

If the ratios between the periods of the individual screen T_j are rational fractions, it is always possible to find the global period comprising of multiple (1, …, N) cells of each of the screens. If not, we can use an iterative strategy to find an approximate global period as explained below. We select an individual FSS, say the j^{th} screen, as the dominant one, whose resonant frequency is the closest to the operating frequency at which we are analyzing the performance of the composite.

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Next, we systematically determine a set of integers L_j ($j=1,....,N$) such that the condition ($T_g=L_j \cdot T_j$) is satisfied to within a certain specified tolerance. We define a relative error E_k as

$$E_{\mathrm{k}} = (T_g - [T_g/ T_k] \cdot T_k)/ T_g, \qquad (1)$$

where $[x]$ implies an integer closest to x. When the value of E_{k} is < 2~3%, for all possible k's, we stop the process and define the parameters of the global system using the values of L_{i}.

2.2. COMPUTATION of SCATTERING MATRIX

At this point, we can follow the obvious path of generating the scattering matrices of the composite using the global period for all the screens. However, this is not a recommended procedure, because it would be highly computer intensive if we wished to maintain the same level of accuracy as we would achieve if we were to use the individual cell sizes of the screens, as well as the same fineness of the cell discretizations as we would when dealing with them when they are isolated.

Furthermore, attempts to compute the Reflection and Transmission Coefficients of the composite directly, *i.e.,* without resorting to *S*-matrices, can become totally impractical if the global period is much larger than the individual ones. In this paper, we introduce a novel technique, described below, that enables us to circumvent both of the problems mentioned above.

To compute the scattering matrix of the composite, we begin by setting the limit on the number of global harmonics to be included in the analysis in accordance with the rules stated in [2]. Next we change the value of the angle of incidence of the plane wave and compute the $[S]_{SI}$ (scattering matrix of the i^{th} subsystem using its own Floquet harmonics, for each of the incident plane waves for which the matrix elements are being generated). We then fill part of the $[S]_{GI}$ (scattering matrix of the i^{th} subsystem expressed in terms of the global harmonics) in accordance with the relationship between the individual and global harmonics as shown in (2). Taking k_x as an example, they are defined by:

$$k_{xs} = \frac{2\pi}{T_{xg}} L_i \bullet m_i + k_{xin0} + m_k \bullet \frac{2\pi}{T_{xg}} = \frac{2\pi}{T_{xg}} (L_i \bullet m_i + m_k) + k_{xin0} \qquad (2)$$

$$m_{ig} = L_i \bullet m_i + m_k$$

where m_i and m_{ig} are incident harmonics of individual and global system, respectively, and m_k is the multiplier of the increment of the incident angle.

2.3 CASCADING FORMULATION

When implementing the cascading procedure for the screens, the speed and the memory requirements are two most important items to consider. This is because there are more than 8 matrices to be generated, and a large number of matrix operations are involved in their computation (see [2]). We find that if we utilize the relationship between the matrices T and R (defined in [2]) as shown in (3), we only need a single matrix inversion. Since this inversion consume a much longer time than does the matrix-matrix multiplication, we can realize a time-saving by using the equations below to compute the S-matrix of the composite.

$$T = [I - S_{22}^{(1)} S_{11}^{(2)}]^{-1}, \qquad R = I + S_{11}^{(2)} T S_{22}^{(1)} \qquad (3)$$

3. NUMERICAL RESULTS

To validate the technique, we consider a commensurate system as our first example (see Fig. 2). For this example, the composite is comprised of three subsystems. The first and the third screens have identical periods (T_1=T_3=2.1 cm), while the period of the second screen T_2=1.05 cm, which is half that of the others. We assume that the FSS elements all have cross-shaped geometries— though these shapes can be totally arbitrary — and the spacing between the FSS screens is 1.0 cm. We choose the number of global harmonic to be 6. Figs. 3 (a) and (b) show the magnitude and phase of the reflection coefficient (transmission coefficient data are not included because of space limitation) of the system. The results obtained from the direct simulation of the entire composite using a CG-FSS algorithm, which is very time-consuming, are also plotted in the above figures for comparison purposes. Figs. 4 (a) and (b) show the magnitudes of the reflection and transmission coefficients for the TE-TE polarization and oblique incidence. We note that the results agree very well to those obtained via the direct simulation.

For the second example, we consider the problem of a non-commensurate system. For this 3-screen stacked composite, T_1=2.1 cm, T_2=1.6 cm and T_3=0.91 cm. The FSS elements of the three screens are again assumed to be cross-shaped, but their sizes are different—with the size of the cross chosen to be proportional to the cell size for each subsystem. The spacing between the FSS screens is 0.75 cm. We identify a global period of T_g=6.3 cm and the period multipliers to be L_1=3, L_2=4 and L_3=7. We generate the results by setting the number of global harmonic at 10, and present the frequency characteristics in Fig. 5. The magnitude of the reflection coefficient of the composite structure and those of each individual screens are plotted in this figure for comparison, for two cases: (a) spacing of 0.75 cm; and (b) 0.4 cm.

It is worthwhile mentioning that for the above example, the CPU times for the direct *vs.* cascade simulations are 99 mins. *vs.* 4.8 mins, respectively, on a Pentium IV PC for a frequency of 12.0 GHz.

4. CONCLUSIONS

In this paper, we have presented an efficient cascading procedure for analyzing an FSS composite system, comprising of multiple FSS screens of unequal periodicities, embedded in multiple dielectric layers. The numerical examples demonstrate the efficiency and effectiveness of the technique. Based on this procedure, a computer program has been developed for the analysis of dissimilar FSS systems, which can be used for both frequency and angular sweep computations.

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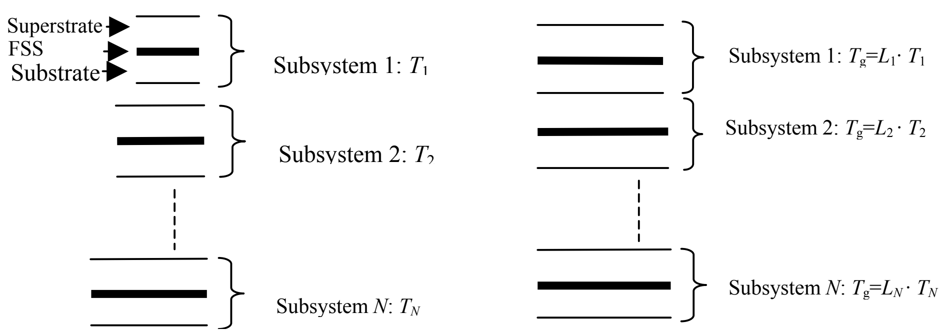


Fig. 1(a) Fig. 1(b)

(a). The composite structure consists of *N* stacked subsystems (side view).

(b). The composite structure in global system (side view).

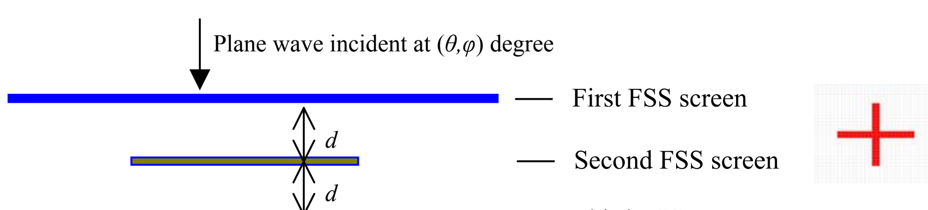
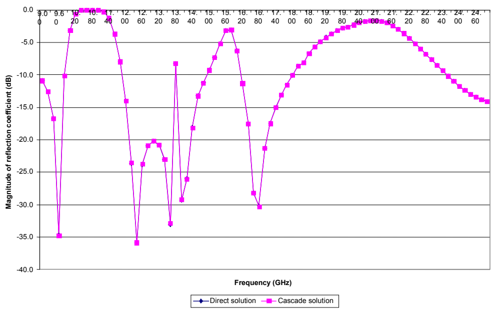


Figure 2. Geometry of a commensurate system and shape of FSS element
First FSS screen: 2.1x 2.1; Second FSS screen: 1.05x1.05; Third FSS screen: 2.1x 2.1; Spacing *d*= 1 (unit: cm).



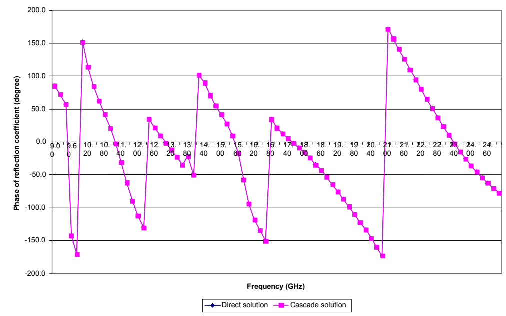
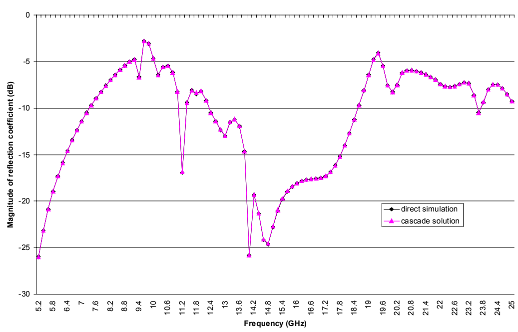


Fig. 3 (a) Fig. 3 (b)

Reflection coefficient (TE-TE polarization) of the composite in Fig. 2 for an incidence angle of (1,1); (a) magnitude in dB; (b) phase in degrees



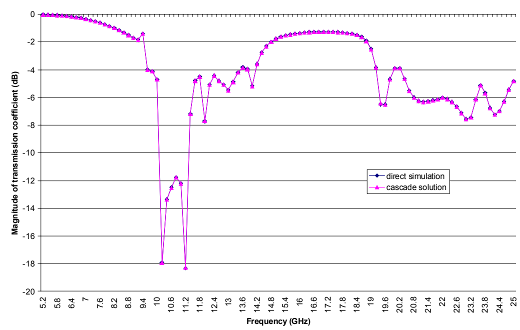
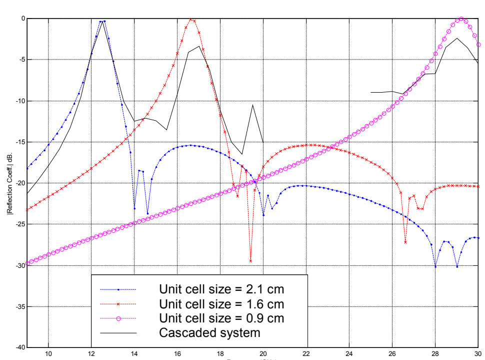


Fig. 4. (a) Fig.4. (b)

Magnitude of (a) reflection and (b) transmission coefficients of the composite in Fig. 2 for an incidence angle of (30, 30) and TE-TE polarization.



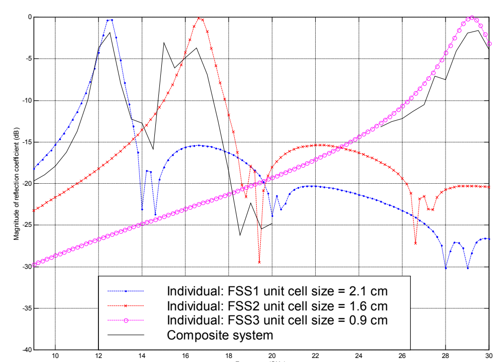


Fig. 5. (a) Fig.5 (b)

Magnitude of reflection coefficient of the second composite system and its comparison with those of the individual screens; (a) spacing *d*=0.75 cm; (b) spacing *d*=0.4 cm