Ionospheric Image Reconstruction Using a Combination of Convex Projections and Regularization Theories

Behzad Sharif, Romina Nikoukar, Farzad Kamalabadi

Department of Electrical and Computer Engineering and Coordinated Science Laboratory, University of Illinois at Urbana-Champaign

Remote sensing measurements of the ionosphere, regardless of the specific physical process under investigation, are often of a line-of-sight integrated nature (e.g., photometric brightness, and TEC). Theory and modeling, as well as monitoring systems, however, are generally concerned with intrinsic volumetric variables, such as wavelength-specific photon volume emission rate or ionospheric electron density. Tomography provides a framework for the estimation of such multi-dimensional parameters from their corresponding one-dimensional observables.

Examples include radio tomography whereby a set of overlapping TEC measurements are utilized, and optical tomography whereby a collection of spectroscopic measurements are inverted to form a multi-dimensional electron density map. The inherent non-ideal acquisition geometry of such remote sensing observations, however, often results in limited-angle tomographic inverse problems that are both ill-posed and ill-conditioned. Furthermore, the intrinsic presence of noise imposes severe challenges on conventional reconstruction methods. To overcome these limitations, we approach the solution of these inverse problems by incorporating a priori information in the form of combined regularization and convex projections.

Regularization theory addresses the limitation of conventional least squares techniques in the presence of noise by replacing a given ill-conditioned problem with a well-conditioned one whose solution is an acceptable approximation to the solution of the original problem. The method of Projection onto Convex Sets (POCS) allows for the incorporation of a priori knowledge in the form of convex constraints, represented formally as:

$$f_{k+1} = P_m P_{m-1} ... P_1 f_k \qquad k = 0, 1, 2, ...$$

where f_{k+1} is the updated image at the kth iteration and P_i is the corresponding projector for the convex constraint set C_i which corresponds to one piece of the *a priori* knowledge of the image to be recovered. Examples of common constraints used in image reconstruction are: image function spatial support, image function amplitude range (e.g., positivity), energy constraint, reference constraint and similarity constraint.

In the proposed method, the two theories are combined in the following manner. At each iteration, first the regularized solution is computed. Next, several convex projections are performed. The first projector, which is usually the positivity projector, takes the regularized image as its input and projects it onto the set of positive images. The projected output is then fed to the next projector. Finally, the output of convex projections is fed to the next iteration of the algorithm which is then used as an initial guess for computing the next regularized image. Experiments show promising results that are superior to conventional Algebraic Reconstruction Technique (ART) and standard regularization methods.