A LOOK AT THE CAUSE OF INSTABILITY PROBLEMS IN FDTD HYBRIDIZATION SCHEMES--AND A PROPOSED REMEDY

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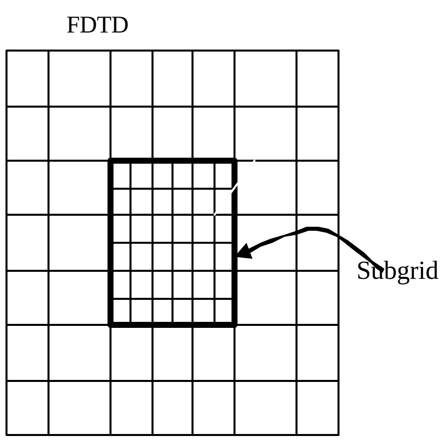
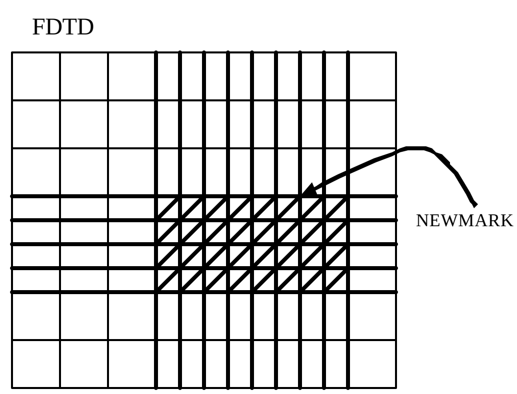
Enforcing the Courant condition on the time step is a reliable approach to guaranteeing, in most part, the stability of the Yee-FDTD algorithm on a uniform or non-uniform but Cartesian grid. However, the same cannot be said about the hybridized version of the FDTD scheme, where in one uses a sub-grid or an unstructured mesh for instance, in a sub-region embedded within the FDTD computational domain. In a recent paper Railton and his co-workers have shown that certain conformal FDTD algorithms suffer from instability problems because they violate reciprocity, and have also suggested a way to rectify the situation by modifying the CFDTD algorithm. However, it is not obvious how a similar analysis can provide a clue as to the origin of the instability problems that frequently arise in hybrid schemes, which combine the conventional FDTD either with explicit sub-gridding algorithms, or with FETD and ADI schemes that are implicit in nature. The problem of instability in the above hybridization schemes has been examined by a number of researchers and it has been observed that the spurious reflections introduced by the interface of the FDTD and the inner sub-region not only corrupts the solution, but also causes the hybrid scheme to be unstable. While the above observations regarding the interface mismatch is indeed correct, it neither provide a convenient indicator for detecting the instability phenomenon in advance of running the FDTD program, nor does it provide a systematic and reliable approach to eradicating the problem.

In this paper we examine the issue of instability on a theoretical basis and show that a systematic indicator, based on the eigenvalues of certain system matrix can serve as a reliable indicator for the instability problem, assuring the stability of the hybrid algorithm if all the eigenvalues are located within the unit circle. This approach not only enables us to establish *a priori* that the algorithm is stable, but also provides us a clue as to how we might attempt to rectify the problem, and re-test the stability using the eigenvalue-spread criterion.

To illustrate the application of approach described above, we consider two scenarios shown in the figures below.





We consider both the cases illustrated above; *viz*., hybridization of coarse grid FDTD scheme with the ADI or the Newmark-beta algorithm, on: (i) fine Cartesian grid; (ii) non-Cartesian grid, such as a triangular one.

The formulation of the stability problem is based on the Cell Method recently introduced by Marrone and his co-workers in a number of recent papers.