One-step algorithm to solve the time-dependent Maxwell equations

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We discuss a unified framework to construct and analyze finite-difference time-domain (FDTD) algorithms to solve the time-dependent Maxwell equations for systems with current sources and spatial variations in both the permittivity and the permeability. The basic idea of this approach is to use (variants of) the Lie-Trotter-Suzuki product-formula to approximate the formal solution, i.e. the time evolution matrix, of the Maxwell equations.

The conventional Yee algorithm naturally fits into this framework and we demonstrate that, by minor modification of the original Yee algorithm, second-order and fourth-order time-accurate schemes can be constructed that do not require the use of staggered-in-time fields, nor extra memory to store intermediate results.

In the case that the energy of the electromagnetic (EM) fields is a conserved quantity, the time evolution matrix is the matrix exponential of a skew-symmetric matrix. Approximations to the time evolution matrix that preserve this symmetry take the form of products of orthogonal transformations. The resulting numerical algorithms are unconditionally stable by construction. Replacing each of the matrix exponentials by its (1,1) Padé approximation (Cayley form) yields the unconditionally stable alternating-direction-implicit time-stepping algorithms proposed by F. Zheng and co-workers and T. Namiki.

In practice, to maintain a reasonable degree of accuracy during the time integration (and stability in the case of Yee-type algorithms), the time step in FDTD calculations has to be relatively small. Then, the amount of computational work required to propagate the EM fields for long times may be prohibitive for a class of important applications, such as bioelectromagnetics and VLSI design. Therefore it is of interest to explore alternative approaches that improve the accuracy and/or efficiency of the time integration.

One such possibility is to approximate the time evolution matrix by a Chebyshev polynomial of the skew-symmetric matrix that represents the (discretized) spatial part of the Maxwell equations. The result of this approach is a one-step algorithm that solves the time-dependent Maxwell equations for (very) large time steps but does not provide information about the EM fields for intermediate times.

Based on results of numerical experiments and rigorous error analysis, we discuss the virtues and shortcomings of the different algorithms. For applications where the long-time behavior is of main interest, we find that the one-step algorithm may be an order of magnitude more efficient than present multiple time-step FDTD algorithms.