Toward the Creation of a Magic Time Step in Three-Dimensional FDTD Algorithms

C. L. Wagner* and J. B. Schneider
School of Electrical Engineering and Computer Science
Washington State University, Pullman, WA 99164-2752, USA
E-mail: clwagner@eecs.wsu.edu, schneidj@eecs.wsu.edu

When operated at the Courant stability limit, the one-dimensional (1D) Yee algorithm is a numerically exact differential equation solver—the only errors are due to the finite precision of digital computer arithmetic. The stability limit in 1D is obtained when the temporal step size is equal to the product of the spatial step size and the speed of propagation. Since this temporal step size yields zero errors, it is termed the "magic" time step. Unfortunately there is no magic time step in the classic Yee algorithm in higher dimensions.

However, for hyperbolic systems of coupled first-order equations (such as Maxwell's equations or the small-signal acoustics equations), volumetric (as opposed to point) gradient, divergence, and curl operators can be derived which allow the construction of an algorithm that has a magic time step in three-dimensions. Theoretically, this magic time step can be arbitrarily large, but the temporal step size is tied to a spatial integration which also increases in size as the temporal step size increases. Other than requiring that fields be staggered in time, the proposed algorithm is not tied to any specific grid. The algorithm requires the calculation of single-variable derivatives of multi-variable integrals of fields but it does not dictate the way in which these calculations are performed. Thus, fields may be offset or collocated. In a computer implementation of the algorithm the underlying calculations use sampling and reconstruction theory where the operators reduce to volumetrically summing the product of weighting coefficients and field values. Although the update coefficients can be precomputed prior to a simulation, obtaining them for any particular space-time grid is computationally expensive.

To demonstrate the method, an approximate version of the proposed algorithm is used to solve a rectangular resonator problem. As will be shown, even this simple (though certainly somewhat brute-force) implementation of the algorithm provides a good prediction of the true resonant frequencies up to the sampling limits imposed by the discretization of time and space. Results are also compared to, and shown to be superior to, those obtained using the standard Yee algorithm. As will be discussed, the computational expense of the global operators used in the algorithm is large since these operators are volumetrically global (as opposed to an algorithm such as PSTD which employs operators which are one-dimensionally global).