An Integral Equation Technique for the Determination of Modal Fields in Arbitrary Cross-section Waveguides

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Several techniques have been applied to the determination of modal fields and cutoff frequencies of arbitrary cross-section waveguides, such as variational methods, finite element method and integral equations. In a typical formulation by integral equations, the metallic walls of the waveguide are replaced by electric current densities in free space, appropriate boundary conditions are applied, and the values of the current densities are calculated by the moment method.

In the present work, it is presented an alternative formulation of the integral equation method, based on dyadic Green's functions of parallel plate waveguides and the moment method. The advantages of the proposed formulation are that it avoids numerical integrations (all integrals are performed analytically) and the modal fields are represented by a single series over orthogonal functions (trigonometric functions). The waveguide under consideration is initially replaced by an equivalent structure where it is circumscribed by a parallel plate waveguide, with horizontal plates, and has its contour approximated by a polygon. It is assumed that a modal wave, with propagation constant β_o, is propagating inside the arbitrary cross-section waveguide. The surface current density induced on the walls of the waveguide is determined by the moment method. TE and TM modes are considered independently. For the case of TM modes, the induced current flows only in the direction of propagation (z direction), and is expanded into a summation over pulse basis functions: $\vec{J} = \sum_i \alpha_i J_i e^{-j\beta_o z} \vec{a}_z$, where J_i are the pulse functions, $\vec{a}_z$ is the unit vector in the z direction, and α_i are coefficients to be determined. The coefficients α_i are computed by the Galerkin's method. In this computation, the electric fields corresponding to each basis function are determined by with the help of the dyadic Green's functions of the parallel plate waveguide. Due to the functional dependence on z of the basis functions and of the dyadic Green's functions, all the necessary integrations can be performed analytically (using the residue theorem), avoiding approximations and numerical integrations inherent to the use of free space Green's functions. In the case of TE modes, it is assumed that $\beta_o = 0$ (cutoff frequency). The magnetic field has only a z component, and the induced surface current density is transverse to the z direction, $\vec{J} = \sum_i \alpha_i J_i e^{-j\beta_o z} \vec{a}_T$, $\vec{a}_T$ being a unit vector perpendicular to $\vec{a}_z$. The moment method is applied to the determination of α_i, considering the boundary condition: $\sum_i k_i H_i = \alpha_j J_j$, here H_i is the magnetic field, due to the ith component of $\vec{J}$, at an observation point on the walls of the waveguide. κ is equal to 0.5 (unless H_i is generated by a vertical current density). This coefficient is necessary since the magnetic fields are expressed as a Fourier series (modal fields of the parallel plate waveguide) of a function with a discontinuity on the surface of the arbitrary cross-section waveguide (the value of the magnetic field changes from J_j to zero). Application of this method to triangular, elliptical and circular waveguides produced good results, with errors less than 0.3% for the first six modes.