A MGCR Multiple RHS Preconditioned Solver Applied to The Resolution of a FE-BI Equation With FMM Acceleration

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Among the various numerical techniques to solve Maxwell Equation, Integral Equations (IE) are one of the most popular and widely applied to study antennas, stealth objects, and the EM Compatibility of systems. The objet mesh is parameterized with N basis functions developing the unknowns electric and/or magnetic currents. The fundamental principal with IE is to evaluate the interaction between each couple of basis functions, leading to a full linear system of size N. With classical algorithm (such as LU), the storage requirement and the CPU time needed to solve the problem scales respectively to $O(N^2)$ and $O(N^3)$. The method is quite insensitive to the number K of right hand sides (RHS) representing the various EM excitations of the object under test, because after factorizing the matrix, the resolution process scales to $O(KN^2)$.

In recent years, the Fast Multilevel Multipole method (FMM) is gaining a wide success in EM computing. The key point is a specific algorithm with allows the evaluation of a matrix vector product scaling to O(NlogN) (it is $O(N^2)$ with standard products). To benefit of FMM speed, one as to replace a classical solver with an iterative solver involving matrix vector products. Unfortunately with multiple RHS, the iterative process as to be restart from beginning with each RHS.

To overcome this problem, we use a GCR (Generalized Conjugate Residual) algorithm because at each iteration the descent vector is evaluated from the residual (with GMRES the residual isn't calculated at each iteration) at there is a explicit perpendicularity of the directions of descent. With a MGCR solver, we solve several RHS simultaneously ; at each iteration the vector of descent is built from a different RHS, for example the one with a maximum residual. With a single matrix-vector product per iteration, MGCR algorithm applied nicely to full matrix, the product been accelerated with FMM. As for any iterative solver a preconditioner can optimize the convergence. In our implementation we use a right side preconditioner.

The MGCR solver has been implemented and tested in two situations : PEC objects and mixed PEC and dielectric objets solved with a FE-BI technique. First, we show that with a single RHS, MGCR converges with a speed equivalent to the widely used GEMRES algorithm. With multiple RHS, various applications illustrate the benefit of MGCR and we obtain a reduction of a factor 6 with MGCR versus independent resolution for each RHS. It has to be noted that this benefit is sensitive to EM nature of the excitations. For example if the excitation is set of plane waves with varying incidences, CPU time reduction is obtained for RHS with the same polarisation (phi or theta). With RHS mixing polarization, the orthogonality of the excitations doesn't lead to any gain and the results are equivalent with 2 separated groups of RHS having same polarisation. Nevertheless, practical application with varying incidences, the MGCR is a quite promising technique.